1.

(a) $\displaystyle\int \frac{\sqrt{3+\sqrt{x}}}{\sqrt{2x}}\,dx$ $u = 3+\sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$

$= \displaystyle\int \sqrt{2}\cdot\sqrt{u}\,du$

$= \frac{2\sqrt{2}}{3}\,u^{3/2} + C$

$= \frac{2\sqrt{2}}{3}\,(3+\sqrt{x})^{3/2} + C$

(b) $\displaystyle\int_0^{\ln(\sqrt{2})} \frac{1+\cos(e^{-2x})}{e^{2x}}\,dx$ $u = e^{-2x}$, $du = -2e^{-2x}dx$

$= \displaystyle\int_1^{1/2} \frac{-1}{2}\,(1+\cos(u))\,du$

$= \frac{-1}{2}\,(u+\sin(u))\Big|_1^{1/2}$

$= \frac{1}{4} - \frac{1}{2}\,(\sin(\tfrac{1}{2}) - \sin(1))$

(c) $\displaystyle\int_0^1 \frac{dx}{\sqrt{e^x}}$

$= \displaystyle\int_0^1 e^{-x/2}\,dx$ $u = \frac{-x}{2}$, $du = \frac{-1}{2}\,dx$

$= \displaystyle\int_0^{-1/2} -2e^u\,du$

$= -2e^u\Big|_0^{-1/2}$

$= 2 - \frac{2}{\sqrt{e}}$

2. $\displaystyle\int \sec^2(x)\cdot\tan(x)\,dx$ $u = \tan(x)$, $du = \sec^2(x)dx$ OR $u = \sec(x)$, $du = \sec(x)\tan(x)dx$

$= \displaystyle\int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\tan^2(x) + C$ OR $\frac{1}{2}\sec^2(x) + C$

The answer's are different, but both are correct. Two antiderivatives differ by a constant, and the identity $\tan^2(x)+1 = \sec^2(x)$ shows that $\frac{1}{2}\tan^2(x) - \frac{1}{2}\sec^2(x) = \frac{-1}{2}$.

L34 P2