Lesson 4: Sections 2.1, 2.2, 2.3

Limits

- $\lim_{x \to a} f(x) =$ the number the outputs get close to when inputs get close to (but not equal to) a
- $\underbrace{\lim_{x \to a^-}}$ $f(x)$ "Left Limit"

  inputs get close to a from the left
  (inputs a little smaller than a)
- $\underbrace{\lim_{x \to a^+}}$ $f(x)$ "Right Limit"

  inputs get close to a from th right
  (inputs a little bigger than a)
- $\lim_{x \to a} f(x)$ might have nothing to do with $f(a)$. These are equal for special functions called continuous functions.

Computing Limits

- $\lim_{x \to a} C = C$ if C is a number
- $\lim_{x \to a} x = a$
- Product Law:

  $$\lim_{x \to a} (f(x) \cdot g(x)) = \left(\lim_{x \to a} f(x)\right) \cdot \left(\lim_{x \to a} g(x)\right)$$
- Quotient Law:

  $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\left(\lim_{x \to a} f(x)\right)}{\left(\lim_{x \to a} g(x)\right)}$$
- Sum/Difference Law:

  $$\lim_{x \to a} (f(x) + g(x)) = \left(\lim_{x \to a} f(x)\right) + \left(\lim_{x \to a} g(x)\right)$$
- Power/Root Law:

  $$\lim_{x \to a} (f(x))^c = \left(\lim_{x \to a} f(x)\right)^c \quad c \text{ a number}$$
- Note: The above laws are true only when all limits exist

Polynomials and Rational Functions

- Polynomials are continuous:
  If $f(x)$ is a polynomial function,
  $$\lim_{x\to a} f(x) = f(a)$$
- Rational functions (polynomial divided by polynomial) are continuous where defined:
  $$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}$$
  if $g(a)$ is defined

Squeeze Theorem

- If $f(x) \le g(x) \le h(x)$ near $x=a$, then
  $$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x) \le \lim_{x\to a} h(x).$$

Practice Problems

1. Find
$$\lim_{x\to -4} \frac{\sqrt{x^2+9} - 5}{x+4}$$

2. If
$$f(x) = \begin{cases} x^2+3 & \text{if } x \le 0 \\ 3-3x & \text{if } 0 < x < 2 \\ 5 & \text{if } x = 2 \\ 2x-4 & \text{if } x > 2 \end{cases}$$

(a) find
$f(0)$
$\lim_{x\to 0^+} f(x)$
$\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0} f(x)$

(b) find
$f(2)$
$\lim_{x\to 2^+} f(x)$
$\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2} f(x)$