Lesson 5: Sections 2.4 and 2.5

Vertical Asymptotes (Infinite Limits)

- Vertical line $x=a$ is a VA of $f(x)$ if

$$\lim_{x\to a^+} f(x) \quad \underline{\text{OR}} \quad \lim_{x\to a^-} f(x)$$

is ∞ or $-\infty$

- Rational functions (polynomial divided by polynomial) have vertical asymptotes at x-values that make the denominator zero after simplifying

Example: $f(x) = \dfrac{x+2}{x^2-4} = \dfrac{x+2}{(x+2)(x-2)}$

Simplifying the expression gives

$$\frac{1}{x-2}$$

The only VA is $x=2$.

- The graph of $f(x)$ looks like a vertical line near a VA

Horizontal Asymptotes (Limits at Infinity)

- Horizontal line $y=a$ is an HA of $f(x)$ if

$$\lim_{x\to -\infty} f(x) = a \quad \underline{\text{OR}} \quad \lim_{x\to\infty} f(x) = a$$

(where a is a number)

- Compute limits at infinity of rational functions by dividing numerator and denominator by the power of the dominant term in the denominator

Example:

$$\lim_{x\to\infty} \frac{x^3+x^2}{\boxed{x^4}+x} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{1}{x^2}}{1+\frac{1}{x^3}}$$

dominant term

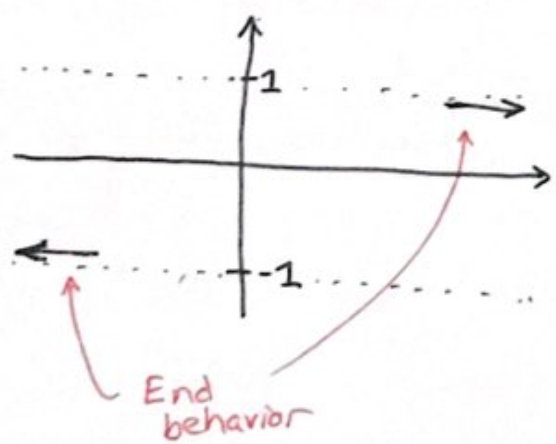
$$= \frac{0}{1}$$

$$= 0$$

So $f(x) = \dfrac{x^3+x^2}{x^4+x}$ has a (right) HA $y=0$.

- HAs encode the "end behavior" of $f(x)$. That is, if $y=a$ is an HA for $f(x)$, then far from the origin the graph of $f(x)$ looks like the horizontal line $y=a$

Example: If $\lim_{x\to-\infty} f(x) = -1$ and $\lim_{x\to\infty} f(x)=1$, the graph has the following end behavior:



Practice Problems

1. (a) Find
$$\lim_{t\to 0^+} \frac{t+4}{t^2+2t}$$
(b) Find
$$\lim_{t\to 0^+} \left(\frac{2}{t} - \frac{t+4}{t^2+2t}\right)$$
(c) What can you say about the symbol $\infty - \infty$?

2. Find
$$\lim_{x\to\infty} \sqrt{x^2-2x} - x$$