Lesson 6: Section 2.6

<u>Note</u>: This semester Section 2.6 <u>will</u> be on Exam 1. Next Tuesday! This is different than the semester the lecture videos were created.

Continuity

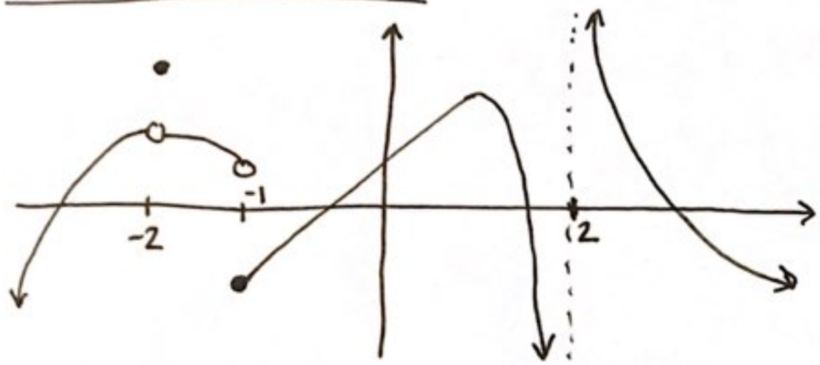
- $f(x)$ is <u>continuous</u> at $x=a$ if:
  1. $f(a)$ is defined
  2. $\lim_{x \to a} f(x)$ exists
  3. $\lim_{x \to a} f(x) = f(a)$

- $f(x)$ is <u>continuous from the left</u> if $\lim_{x \to a^-} f(x) = f(a)$ and <u>continuous from the right</u> if $\lim_{x \to a^+} f(x) = f(a)$.

- $f(x)$ is continuous **on** $[-4,3)$ means continuous from the right at -4 and <u>not</u> continuous from the left at 3

- Functions continuous on their domain (AKA, where they are defined):
  - polynomials
  - rational functions
  - root functions
  - trigonometric functions
  - exponential functions
  - logarithmic functions.

Discontinuities



- Hole at $x=-2$: $\lim_{x \to -2} f(x)$ exists but $f(-2)$ either not defined or $\lim_{x \to -2} f(x) \neq f(-2)$
- Jump at $x=-1$: $\lim_{x \to -1^-} f(x)$ and $\lim_{x \to -1^+} f(x)$ are different numbers
- Infinite Discontinuity at $x=2$: $\lim_{x \to 2^-} f(x)$ or $\lim_{x \to 2^+} f(x)$ are ∞ or $-\infty$

Intermediate Value Theorem

- If $f(x)$ is continuous on the interval $[a,b]$, then $f(x)$ takes on every value between $f(a)$ and $f(b)$.

Practice Problems

1. Individually, sketch a function which goes through the points $(0,1)$ and $(2,4)$, but does not cross the line $y=3$ on the interval $(0,2)$. Share with your group. What do your functions have in common? Why?
2. Find a and b so that $f(x)$ is continuous at all x:

$$f(x) = \begin{cases} x^2 & \text{if } x<1 \\ ax+b & \text{if } 1 \le x \le 2 \\ \frac{x^2+1}{x-1} & \text{if } x>2 \end{cases}$$