

1. Graphs will vary, but cannot be continuous on the interval $[0, 2]$.

If the graph was continuous on $[0, 2]$, the intermediate value theorem tells us there is some c between 0 and 2 with $f(c) = 3$.

2.

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ ax+b & \text{if } 1 \leq x \leq 2 \\ \frac{x^2+1}{x-1} & \text{if } x > 2 \end{cases}$$

Since each piece-wise component of $f(x)$ is continuous on its domain, we need only check continuity at $x=1$ and $x=2$.

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 = 1$$
$$f(1) = a + b$$
$$\left. \begin{array}{l} \lim_{x \rightarrow 1^-} f(x) = 1 \\ f(1) = a + b \end{array} \right\} \text{so: } a+b=1$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{x^2+1}{x-1} = 5$$
$$f(2) = 2a+b$$
$$\left. \begin{array}{l} \lim_{x \rightarrow 2^+} f(x) = 5 \\ f(2) = 2a+b \end{array} \right\} \text{so: } 2a+b=5$$

$$\begin{array}{r} 2a+b = 5 \\ -(a+b = 1) \\ \hline a = 4 \end{array}$$

$$\text{and } b = 1 - a = -3.$$