Lesson 7: Sections 3.1, 3.2

Definition of the Derivative

- The secant lines through a point approach the tangent line at that point. See

https://www.desmos.com/calculator/zr7siihtmz

- The derivative of $f(x)$ gives the slope of the tangent line at $x=a$, and can be defined in two ways:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

The Derivative as a Function

- $f(x)$ is not differentiable at $x=a$ if:
  - $f(x)$ is not continuous at $x=a$
  - $f(x)$ has a vertical tangent at $x=a$
  - $f(x)$ is not smooth at $x=a$
- We can find the function $f'(x)$ using the second definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- The derivative gives information about the graph of $f(x)$:

$f'(x)$ positive $\iff$ $f(x)$ increasing

$f'(x)$ negative $\iff$ $f(x)$ decreasing

$f'(x)$ is zero $\iff$ $f(x)$ has a horizontal tangent

Practice Problems

1. Individually, draw a function $f(x)$ (not $f'(x)$) with the following properties:

$$\begin{cases} f'(x)=0 & \text{at } x=1,4 \\ f'(x) & \text{undefined at } x=3 \\ f'(x)>0 & \text{on } (-\infty,1)\cup(1,3)\cup(3,4) \\ f'(x)=-x+4 & \text{on } (4,\infty) \end{cases}$$

Discuss your graphs with your group. What do your graphs have in common? What is different?

2. Use the limit definition to find the derivative of $f(x)=x^{1/3}$ at $x=0$, if it exists. Also find the equation of the tangent line to $f(x)$ at $x=0$, if it exists.

3. Show $f(x)=|x|$ is <u>not</u> differentiable at $x=0$ by using the limit definition of the derivative