

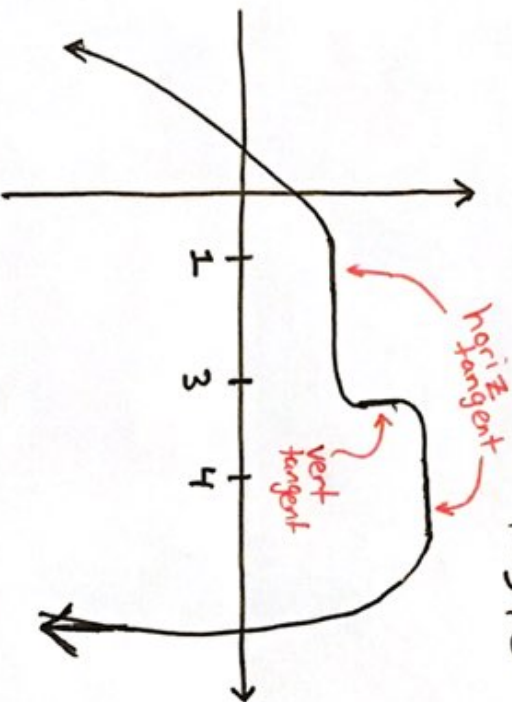
1.

$f'(x)=0 \iff f(x)$ has a horizontal tangent

$f'(x)$ undefined $\iff f(x)$ not continuous, has a vertical tangent, or is not smooth

$f'(x) > 0 \iff f(x)$ increasing

$f'(x) = -x+4 \iff$ larger and larger x -values make $f(x)$ decrease faster and faster



L7 P3

2.

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - 0}{x - 0}$$

$$= \lim_{x \rightarrow 0} \frac{x^{1/3}}{x}$$

$$= \lim_{x \rightarrow 0} x^{-2/3} = \frac{1}{\sqrt[3]{x^2}}$$

$$= \infty$$

So $f'(0)$ is not defined.

The tangent line to $f(x) = x^{1/3}$ at $x=0$ is the vertical line $x=0$ since the limit is infinite.

$$3. \lim_{x \rightarrow 0^-} \frac{f(x) - 0}{x - 0} = \lim_{x \rightarrow 0^-} \frac{|x|}{x} = \lim_{x \rightarrow 0^-} \frac{-x}{x} = -1$$

$$\lim_{x \rightarrow 0^+} \frac{f(x) - 0}{x - 0} = \lim_{x \rightarrow 0^+} \frac{|x|}{x} = \lim_{x \rightarrow 0^+} \frac{x}{x} = 1$$

Since the left and right limits disagree, $\lim_{x \rightarrow 0} \frac{f(x) - 0}{x - 0}$ DNE.