Lesson 8: Section 3.3

Notation:

- $\frac{d}{dx}(\_\_\_)$ and $(\_\_\_)'$ both mean "the derivative of $\_\_\_$"
- If $y = f(x)$:

  1ST Deriv: y' OR $f'(x)$ OR $\frac{dy}{dx}$ OR $\frac{df}{dx}$

  2ND Deriv: y'' OR $f''(x)$ OR $\frac{d^2y}{dx^2}$ OR $\frac{d^2f}{dx^2}$

  3RD Deriv: $y^{(3)}$ OR $f^{(3)}(x)$ OR $\frac{d^3y}{dx^3}$ OR $\frac{d^3f}{dx^3}$

  $\vdots$

Basic Rules of Differentiation

- Power Rule

$$\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$$

- Constant Multiple Rule

$$\frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot \frac{d}{dx}\left(f(x)\right)$$

- Sum/Difference Rules

$$\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx}\left(f(x)\right) + \frac{d}{dx}\left(g(x)\right)$$

$$\frac{d}{dx}\left(f(x) - g(x)\right) = \frac{d}{dx}\left(f(x)\right) - \frac{d}{dx}\left(g(x)\right)$$

Caution:

- $\frac{d}{dx}\left(f(x) \cdot g(x)\right) \neq \frac{d}{dx}\left(f(x)\right) \cdot \frac{d}{dx}\left(g(x)\right)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) \neq \frac{\frac{d}{dx}\left(f(x)\right)}{\frac{d}{dx}\left(g(x)\right)}$

The Natural Exponential Function

- Define the number
  $e = 2.71828\ldots$
  by the property that if $f(x) = e^x$, then $f'(0) = 1$. This is called Euler's number.

- The inverse of the natural exponential function $f(x) = e^x$ is the natural log function
  $$f^{-1}(x) = \log_e(x) = \ln(x)$$

- $$\frac{d}{dx}\left(e^x\right) = e^x$$

Practice Problems

1. (a) $\frac{d^2}{dx^2}\left(x^4\sqrt[3]{x^2}\right) =$

   (b) $\frac{d^2}{dx^2}\left(\frac{x^3+7x+5}{x\sqrt{x}}\right) =$

   (c) $\frac{d^2}{dx^2}\left[e^x + x^e\right] =$

2. Find conditions on numbers a, b, c, and d so that
   $$f(x) = ax^3 + bx^2 + cx + d$$
   has exactly
   (a) two horizontal tangents
   (b) one horizontal tangent
   (c) zero horizontal tangents