

1. (a)

$$\frac{d^2}{dx^2} \left(x^4 \sqrt{x^2} \right) = \frac{d^2}{dx^2} \left(x^{14/3} \right)$$

$$= \frac{d}{dx} \left(\frac{14}{3} x^{11/3} \right)$$

$$= \frac{154}{9} x^{8/3}$$

$$(b) \frac{d^2}{dx^2} \left(\frac{x^3 + 7x + 5}{x\sqrt{x}} \right) = \frac{d^2}{dx^2} \left(x^{3/2} + 7x^{-1/2} + 5x^{-3/2} \right)$$

$$= \frac{d}{dx} \left(\frac{3}{2} x^{1/2} - \frac{7}{2} x^{-3/2} - \frac{15}{2} x^{-5/2} \right)$$

$$= \frac{3}{4} x^{-1/2} + \frac{21}{4} x^{-5/2} + \frac{75}{4} x^{-7/2}$$

$$(c) \frac{d^2}{dx^2} (e^x + x^x) = \frac{d}{dx} (e^x + e^x x^{x-1})$$

$$= e^x + e(e-1)x^{e-2}$$

2. $f(x) = ax^3 + bx^2 + cx + d$

has a horizontal tangent when $f'(x) = 0$.

$$f'(x) = 3ax^2 + 2bx + c$$

The quadratic formula says the solutions to $f'(x) = 0$ are

$$x = \frac{-2b \pm \sqrt{4b^2 - 4ac}}{6a}$$

There are two solutions if $4b^2 - 12ac > 0$, one solution if $4b^2 - 12ac = 0$, and zero solutions if $4b^2 - 12ac < 0$.