Lesson 9: Section 3.4

Product Rule:

- $\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$
- Since there is a "+", the order doesn't matter:

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = g'(x)f(x) + f'(x)\cdot g(x)$$

- "Derivative of the first times the second left alone plus the first left alone times the derivative of the second"

- You can take the derivative of a product of three or more functions by grouping and using the usual product rule. The outcome follows the same pattern: each function gets one turn as a derivative.

Example:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\cdot h(x)\right)$$

$$= f'(x)\cdot g(x)\cdot h(x) + f(x)\cdot g'(x)\cdot h(x) + f(x)\cdot g(x)\cdot h'(x)$$

Quotient Rule

- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$
- Since there is a "$-$", the order <u>does</u> matter.
- $\frac{d}{dx}\left[\frac{\text{high}}{\text{low}}\right] = \frac{\text{low}\cdot(\text{d-high}) - \text{high}\cdot(\text{d-low})}{(\text{low})(\text{low})}$

"Low D High minus High D Low over the square of what's below"

<u>Note:</u> Sometimes simplifying and using the power rule is easier than using the product/quotient rules.

<u>Example:</u> $\frac{d}{dx}\left[x\cdot\sqrt{x}\cdot x^{2/3}\right]$

$\frac{d}{dx}\left[\frac{x^2+1}{x}\right]$

Practice Problems

1. (Spring 2017 Exam 2 Problem 5) Find all values of x for which the graph of $f(x) = \frac{2x-1}{x+1}$ has a tangent line parallel to the line $y = 3x+1$.

2. Find $\frac{d^2y}{dx^2}$, where

$$y = (5x^2-3)(7x^3+x),$$

in two ways:

(i) using the product rule

(ii) expanding and using the power rule