

Name: _____

ID number: _____

Instructions:

1. This is a one-hour exam. If you continue to work on this exam after time is called it will be considered cheating and you will receive a zero.
2. There are 11 problems on this exam.
3. No books, notes, or calculators are allowed. Only a writing utensil, eraser, and water are allowed on your desk with the exam.
4. Turn off your cell phone.
5. Circle **one and only one choice** for each multiple-choice problem. Showing your work is NOT necessary for multiple-choice problems and no partial credit will be given for multiple-choice problems.
6. Legibly show all relevant work on non-multiple-choice problems. Partial credit will be given for steps leading to the correct solutions.

Little or no work with a correct answer will receive little or no credit.

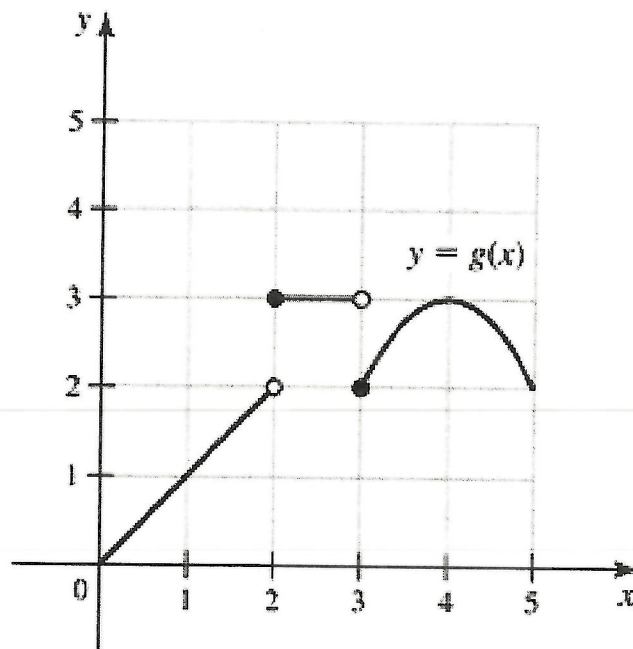
Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

I agree to abide by the instructions above and have read and understood the above statement regarding academic integrity:

Signature: _____

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1. Use the graph of g below to answer the following questions.



- i. (5 points) Evaluate $\lim_{x \rightarrow 3} g(x)$
- A. 2
 - B. 3
 - C. The limit does not exist because g is not differentiable at $x = 3$.
 - D. The limit does not exist because g has a jump discontinuity at $x = 3$.
 - E. The limit does not exist because g has a removable discontinuity at $x = 3$.

- ii. (5 points) Evaluate $\lim_{x \rightarrow 2^-} g(x)$.

- A. 0
- B. 2
- C. 3
- D. The limit does not exist because g is not differentiable at $x = 2$.
- E. The limit does not exist because g has a jump discontinuity at $x = 2$.

2. (8 points) Compute the following limit. *Be sure to show all your work.*

$$\lim_{x \rightarrow 1} \frac{\sqrt{10x-9}-1}{x-1}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{10x-9}-1}{x-1} \cdot \frac{\sqrt{10x-9}+1}{\sqrt{10x-9}+1}$$

$$= \lim_{x \rightarrow 1} \frac{(10x-9)-1}{(x-1)(\sqrt{10x-9}+1)}$$

$$= \lim_{x \rightarrow 1} \frac{10(x-1)}{(x-1)(\sqrt{10x-9}+1)} = \lim_{x \rightarrow 1} \frac{10}{\sqrt{10x-9}+1}$$

$$\boxed{5}$$

3. (8 points) Find all the vertical and horizontal asymptotes of the following function:

$$f(x) = \frac{x^2 - 8x + 16}{5x^2 - 35x + 60} = \frac{(x-4)^2}{5(x-4)(x-3)} = \frac{(x-4)}{5(x-3)}$$

(Write NONE if the function has no asymptote of the given type.)

VA: denominator = 0

HA: $\lim_{x \rightarrow \pm\infty} f(x)$

Vertical asymptote(s):

$$\boxed{x = 3}$$

Horizontal asymptote(s):

$$\boxed{y = 1/5}$$

$$f(x) = \frac{x-4}{5(x-3)}, x \neq 4$$

4. (8 points) Let

$$g(x) = \begin{cases} x^2 + x & \text{if } x \leq 1 \\ ax + b & \text{if } 1 < x < 3 \\ 3x & \text{if } x \geq 3 \end{cases}$$

Determine the values of a and b so that $g(x)$ is continuous for all real numbers.

Make it continuous at $x=1$ and $x=3$

$$\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^+} g(x)$$

$$\lim_{x \rightarrow 3^-} g(x) = \lim_{x \rightarrow 3^+} g(x)$$

$$2 = a + b$$

$$3a + b = 9$$

$$2a = 7$$

$$a = \boxed{7/2}$$

$$b = 2 - \frac{7}{2} = -\frac{3}{2}$$

$$b = \boxed{-3/2}$$

5. (10 points)

i. Write down one of the definitions of the derivative of a function f at a number a .

5

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \quad \text{OR} \quad \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

5

ii. Compute the derivative of $f(x) = \frac{1}{x+5}$ at the number $a = 5$ using the definition you wrote above.

$$\lim_{x \rightarrow 5} \frac{\frac{1}{x+5} - \frac{1}{10}}{x-5}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{10+h} - \frac{1}{10}}{h}$$

$$= \lim_{x \rightarrow 5} \frac{1}{x-5} \cdot \left(\frac{10-x-5}{(x+5)10} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{10-10-h}{10(10+h)} \right)$$

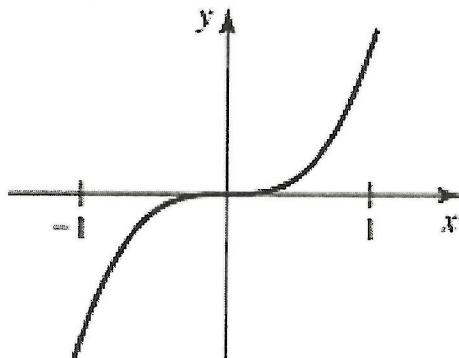
$$= \lim_{x \rightarrow 5} \frac{-x+5}{(x-5)(x+5)(10)}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{10(10+h)}$$

$$\boxed{-\frac{1}{100}}$$

$$= \lim_{x \rightarrow 5} \frac{-1}{x+5(10)}$$

6. The graph of $f(x)$ is given by the following figure.

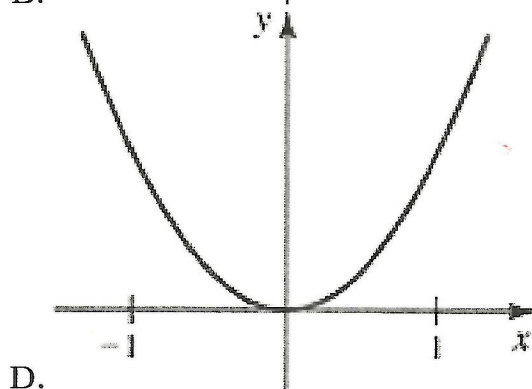
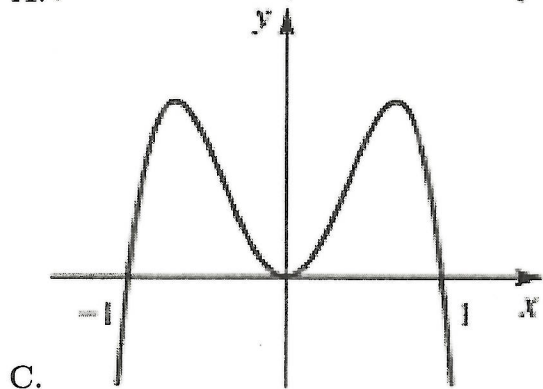
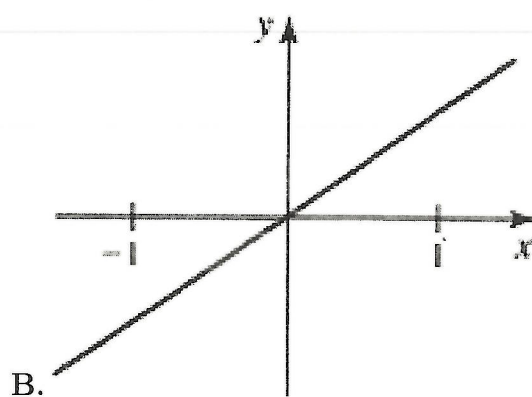
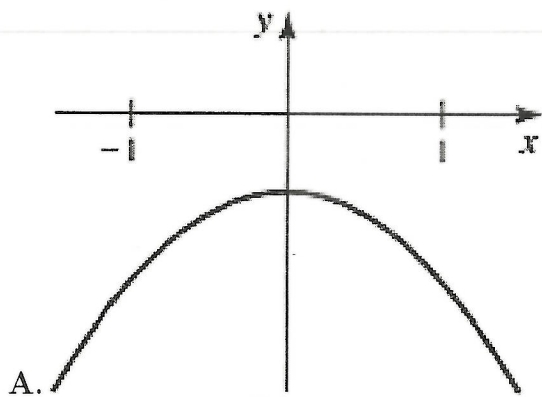


i. (5 points) Which of the below could be the derivative of f ? Circle one

A B C **D** E

ii. (5 points) Which of the below could be the second derivative of f ? Circle one

A **B** C D E



E. None of the above

7. Let $f(x) = (x-1)e^x - 6x^2$.

i. (5 points) Find all points on the graph of f at which the tangent line is horizontal.

horizontal tangent line \Leftrightarrow where $f'(x) = 0$ ✓

$$f'(x) = e^x + (x-1)e^x - 12x = xe^x - 12x$$

$$f'(x) = 0 \text{ when } xe^x - 12x = 0$$

$$x(e^x - 12) = 0$$

$$x=0, \ln 12$$

ii. (5 points) Find $f''(x)$.

$$f''(x) = e^x + xe^x - 12$$

OR

$$= (x+1)e^x - 12$$

$$(x+1)e^x - 12$$

8. (8 points) Let $h(r) = \frac{2-r-\sqrt{r}}{r+1}$.

Find the equation of the line tangent to the graph of h at $(4, -\frac{4}{5})$.

equation of the tangent line: $y - h(4) = h'(4)(x - 4)$

$$h'(r) = \frac{(r+1)(-1 - \frac{1}{2\sqrt{r}}) - (2-r-\sqrt{r})}{(r+1)^2}$$

$$h'(4) = \frac{5(-1 - \frac{1}{4}) - (2-4-2)}{5^2}$$

$$= \frac{5(-\frac{5}{4}) + 4}{25}$$

$$= \frac{1}{25}(-\frac{25}{4} + \frac{16}{4})$$

$$= -\frac{9}{100}$$

$$y + \frac{4}{5} = -\frac{9}{100}(x-4)$$

$$y = -\frac{9}{100}x - \frac{11}{25}$$

9. (6 points) Evaluate $\lim_{x \rightarrow 0} \frac{\sin(7x)}{\sin(3x)}$. Show your work.

Use $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{7x} \cdot \frac{3x}{\sin 3x} \cdot \frac{7x}{3x}$$

$$= 1 \cdot 1 \cdot \frac{7}{3}$$

$$\frac{7}{3}$$

10. Suppose the position of an object moving horizontally along a line after t seconds is given by the function:

$$s = 2t^3 - 21t^2 + 60t$$

for $0 \leq t \leq 6$, where s is measured in feet, with $s > 0$ corresponding to positions right of the origin.

- i. (4 points) Find the velocity and acceleration functions.

$$v = s' = 6t^2 - 42t + 60$$

$$a = v' = 12t - 42$$

Velocity:

$$6t^2 - 42t + 60$$

Acceleration:

$$12t - 42$$

- ii. (4 points) When is the object stationary? *Be sure to include the correct units.*

$$\text{stationary} \Leftrightarrow v = 0$$

$$6t^2 - 42t + 60 = 0$$

$$6(t^2 - 7t + 10) = 0$$

$$6(t-2)(t-5) = 0$$

$$t = 2s$$

$$t = 5s$$

- iii. (4 points) What is the acceleration of the object when its velocity is zero? *Be sure to include the correct units.*

$$a(2) = -18 \text{ ft/s}^2$$

$$a(5) = 18 \text{ ft/s}^2$$

$$-18 \text{ ft/s}^2$$

$$18 \text{ ft/s}^2$$

11. Let $h(x) = f(g(x))$ and $k(x) = g(g(x))$.

Use the table to compute the following derivatives:

x	1	2	3	4	5
$f(x)$	0	3	5	1	0
$f'(x)$	-6	-3	8	7	2
$g(x)$	4	1	5	2	3
$g'(x)$	9	7	3	-1	-5

i. (5 points) What is $h'(3)$?

- A. 2
- B. 5
- C. 6
- D. 8
- E. 10

$$\begin{aligned}h'(3) &= f'(g(3)) \cdot g'(3) \\ &= f'(5) \cdot 3 \\ &= 2 \cdot 3 = 6\end{aligned}$$

ii. (5 points) What is $k'(5)$?

- A. -25
- B. -15
- C. 3
- D. 15
- E. 25

$$\begin{aligned}k'(5) &= g'(g(5)) \cdot g'(5) \\ &= g'(3) \cdot (-5) \\ &= 3 \cdot (-5) \\ &= -15\end{aligned}$$