MA 16100                    Midterm Exam 2                    Summer 2019

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Instructions:

1. This is a one-hour exam. If you continue to work on this exam after time is called it will be considered cheating and you will receive a zero.

2. There are 11 problems on this exam.

3. No books, notes, or calculators are allowed. Only a writing utensil, eraser, and water are allowed on your desk with the exam.

4. Turn off your cell phone.

5. Circle **one and only one choice** for each multiple-choice problem. Showing your work is NOT necessary for multiple-choice problems and no partial credit will be given for multiple-choice problems.

6. Legibly show all relevant work on non-multiple-choice problems. Partial credit will be given for steps leading to the correct solutions.
   ***Little or no work with a correct answer will receive little or no credit.***

I agree to abide by the instructions above and have read and understood the above statement regarding academic integrity:

**Signature:** ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

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**1.** (8 points) Find the equation of the line tangent to the curve, $\sin(x - y) + x^3y = y + 14$, at the point $(2, 2)$.

**2.** (10 points) Evaluate $y''(1)$ where $y = e^x + x^e$.

     A. 0

     B. 1

     C. 2

     D. $e$

     E. $e^2$

**3.** (8 points) Evaluate the derivative of the _inverse_ of $f(x) = x^3 - x + 1$ at the point $(1, -1)$.

**4.** (8 points) The graph of $y = x^{\ln(x)+1}$, $x > 0$, has exactly one horizontal tangent line. Find the equation of the horizontal tangent line.

**5.** Evaluate the derivatives of the following functions:

    i. (4 points) Evaluate the derivative of $g(z) = \tan^{-1}(1/z)$ at $z = 1$.

    ii. (4 points) Evaluate the derivative of $h(z) = \dfrac{1}{\cos^{-1}(z)}$ at $z = 0$.

**6.** (8 points) The length of a rectangle is increasing at a rate of 7 cm/s and its width is increasing at a rate of 3 cm/s.

When the length is 12 cm and the width is 8 cm, how fast is the area of the rectangle increasing?

cm/s

**7.** (9 points) An observer stands 300 ft from the launch site of a hot-air balloon at an elevation equal to the elevation of the launch site. The balloon is launched vertically and maintains a constant upward velocity of 20 ft/s.

What is the rate of change of the angle of elevation of the balloon when it is 400 ft from the ground?

rad/s

**8.** (8 points) Determine the value(s) of the absolute extreme values of
$f(t) = \frac{3t}{t^2+1}$ on the interval $[-2, 0]$.

Absolute minimum value:

Absolute maximum value:

**9.** (10 points) Which of the following functions satisfy the conditions of Rolle's Theorem on the interval $[-1, 1]$?

$$f(x) = 1 - x^{2/3}$$
$$g(x) = x^3 - 2x^2 - x + 2$$
$$h(x) = \cos\left(\frac{\pi}{4}(x + 1)\right)$$

Rolle's Theorem applies to:

A. both $f$ and $g$

B. both $g$ and $h$

C. $g$ only

D. $h$ only

E. All three

**10.**    i. (3 points) Fill in the blanks to complete the statement of the Mean Value Theorem:

If $f$ is _____ on $[a, b]$ and _____
on $(a, b)$, then there is at least one point $c$ in $(a, b)$ such that:

$$f'(c) = \frac{\phantom{xxxxxxxxxx}}{b - a}$$

ii. (6 points) The $f(x) = x + \frac{1}{x}$ on $[1, 3]$ satisfies the conditions of the Mean Value Theorem. Find the point(s), $c$, that is/are guaranteed to exist by the Mean Value Theorem.

**11.** Suppose that the derivative of a function $h$ is given by:

$$h'(x) = x(x-1)^2(x-2)$$

   i. (7 points) On what interval(s) is $h$ increasing?

     A. $(-\infty, 0)$

     B. $(-\infty, 0)$ and $(2, \infty)$

     C. $(0, 2)$

     D. $(0, 1)$ and $(2, \infty)$

     E. $(-\infty, 1)$

   ii. (7 points) Where does $h$ have a local maximum?

     A. $x = 0$

     B. $x = 2$

     C. $x = 0$ and $1$

     D. $x = 0$ and $2$

     E. $x = 1$ and $2$