

Name: Key

ID number: \_\_\_\_\_

## Instructions:

1. This is a one-hour exam. If you continue to work on this exam after time is called it will be considered cheating and you will receive a zero.
2. There are 11 problems on this exam.
3. No books, notes, or calculators are allowed. Only a writing utensil, eraser, and water are allowed on your desk with the exam.
4. Turn off your cell phone.
5. Circle **one and only one choice** for each multiple-choice problem. Showing your work is NOT necessary for multiple-choice problems and no partial credit will be given for multiple-choice problems.
6. Legibly show all relevant work on non-multiple-choice problems. Partial credit will be given for steps leading to the correct solutions.

***Little or no work with a correct answer will receive little or no credit.***

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

I agree to abide by the instructions above and have read and understood the above statement regarding academic integrity:

Signature: \_\_\_\_\_

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1. (8 points) Find the equation of the line tangent to the curve,  $\sin(x - y) + x^3y = y + 14$ , at the point  $(2, 2)$ .

Point:  $(2, 2)$  ✓

Slope:  $y'(2)$  ✓

$$\frac{d}{dx} [\sin(x-y) + x^3y] = \frac{d}{dx} [y + 14] \quad \checkmark$$

✓ for each side

$$(\cos(x-y) \cdot (1-y')) + (3x^2y + x^3y') = y' \quad \checkmark$$

$$[-\cos(x-y) + x^3 - 1]y' = -\cos(x-y) - 3x^2y \quad \checkmark$$

$$(-\cos(0) + 2^3 - 1)y' = -\cos(0) - 3 \cdot 2^3 \quad \text{Plug in } (2, 2) \quad \checkmark$$

$$(-1 + 8 - 1)y' = -1 - 24$$

$$y' = \frac{-25}{6} \quad \checkmark$$

$$y - 2 = \frac{-25}{6}(x - 2) \quad \checkmark \checkmark$$

$$y = \frac{-25}{6}x + \frac{50}{6} + 2$$

$$\frac{74}{6} + \frac{31}{3}$$

2. (10 points) Evaluate  $y''(1)$  where  $y = e^x + x^e$ .

A. 0

B. 1

C. 2

D.  $e$ E.  $e^2$ 

$$y' = e^x + e x^{e-1}$$

$$y'' = e^x + e \cdot (e-1) x^{e-2}$$

$$y'' = e + e^2 - e = e^2$$

3. (8 points) Evaluate the derivative of the inverse of  $f(x) = x^3 - x + 1$  at the point  $(1, -1)$ .

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}$$

$$(f^{-1})'(1) = \frac{1}{f'(-1)}$$

$$= \frac{1}{2}$$

$$f'(x) = 3x^2 - 1$$

$$f'(-1) = 3 - 1 = 2$$

$$\boxed{\frac{1}{2}}$$

4. (8 points) The graph of  $y = x^{\ln(x)+1}$ ,  $x > 0$ , has exactly one horizontal tangent line. Find the equation of the horizontal tangent line.

Use logarithmic differentiation ✓  
 Horizontal tangent line:  $y' = 0$  ✓

$$\ln y = (\ln x + 1) \cdot \ln x \quad \checkmark$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} [(\ln x + 1) \ln x]$$

$$\frac{y'}{y} \quad \checkmark = \frac{1}{x} \ln x + \frac{1}{x} (\ln x + 1) \quad \checkmark$$

$$y' = \overset{\text{or } y}{x^{\ln x + 1}} \cdot \frac{1}{x} (2 \ln x + 1) \quad \checkmark \quad \underline{6}$$

Equal to zero when  $2 \ln x + 1 = 0$  ✓ 7 points

$$\ln x = -\frac{1}{2} \quad \checkmark$$

$$x = e^{-1/2} \quad \checkmark$$

$$y = (e^{-1/2})^{\ln(e^{-1/2}) + 1} \quad \checkmark \leftarrow \text{full credit}$$

$$y = (e^{-1/2})^{1/2}$$

$$y = e^{-1/4} \quad \checkmark$$

5. Evaluate the derivatives of the following functions:

i. (4 points) Evaluate the derivative of  $g(z) = \tan^{-1}(1/z)$  at  $z = 1$ .

$$g'(z) = \frac{1}{1+(\frac{1}{z})^2} \cdot (-\frac{1}{z^2}) = -\frac{1}{1+z^2}$$

$$g'(1) = \frac{1}{1+1} \cdot (-1) = -\frac{1}{2}$$

$$-\frac{1}{2} \checkmark$$

ii. (4 points) Evaluate the derivative of  $h(z) = \frac{1}{\cos^{-1}(z)}$  at  $z = 0$ .

$$h(z) = (\cos^{-1}(z))^{-1}$$

$$h'(z) = -(\cos^{-1}(z))^{-2} \cdot \frac{-1}{\sqrt{1-z^2}}$$

$$h'(0) = -(\cos^{-1}(0))^{-2} \cdot \frac{-1}{1}$$

$$= -\left(\frac{\pi}{2}\right)^{-2} \cdot -1$$

$$h'(z) = \frac{1/\sqrt{1-z^2}}{(\cos^{-1}(z))^2}$$

$$h'(0) = \frac{+1}{(\cos^{-1}(0))^2}$$

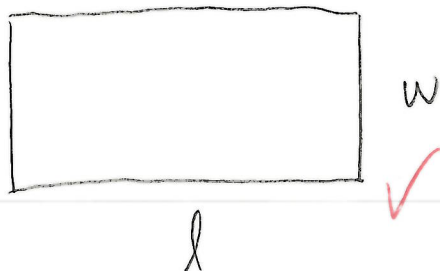
$$h'(0) = \frac{1}{(\frac{\pi}{2})^2}$$

$$\frac{4}{\pi^2} \checkmark$$

6. (8 points) The length of a rectangle is increasing at a rate of 7 cm/s and its width is increasing at a rate of 3 cm/s.

When the length is 12 cm and the width is 8 cm, how fast is the area of the rectangle increasing?

Diagram: ✓  
2pts



3-5 pts for  
set up

Relation: Area  $A = lw$  ✓

Rates:  $\frac{dl}{dt} = 7 \frac{\text{cm}}{\text{s}}$  ✓;  $\frac{dw}{dt} = 3 \frac{\text{cm}}{\text{s}}$  ✓

Rate Relation:  $A' = w \frac{dl}{dt} + l \frac{dw}{dt} = 7w + 3l$  ✓

when  $l = 12 \text{ cm}$  and  $w = 8 \text{ cm}$ ,

$$A' = 7 \cdot 8 + 3 \cdot 12 = 56 + 36$$

$$A' = 92 \text{ cm/s}$$

92 ✓

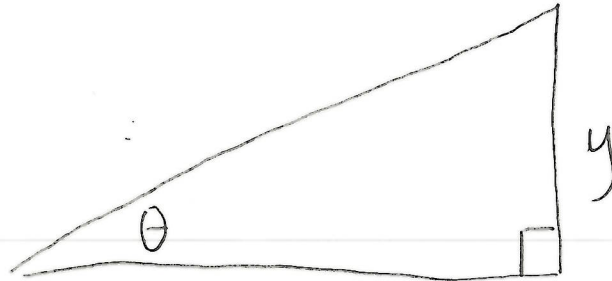
cm/s



7. (9 points) An observer stands 300 ft from the launch site of a hot-air balloon at an elevation equal to the elevation of the launch site. The balloon is launched vertically and maintains a constant upward velocity of 20 ft/s.

What is the rate of change of the angle of elevation of the balloon when it is 400 ft from the ground?

Diagram: ✓  
Labels: ✓



Relation:  $\tan \theta = \frac{y}{300}$  or  $\theta = \tan^{-1}\left(\frac{y}{300}\right)$  ✓

Rates:  $\frac{dy}{dt} = 20$  ft/s ✓

Rate Relation:  $\sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{1}{300} \frac{dy}{dt}$  or  $\frac{d\theta}{dt} = \frac{1}{1 + (y/300)^2} \cdot \frac{dy}{dt} \cdot \frac{1}{300}$

Plug in:  $y = 400$  (then  $z = 500$ ) ✓

$$\left(\frac{500}{300}\right)^2 \frac{d\theta}{dt} = \frac{1}{300} \cdot 20$$

$$\begin{aligned} \text{or } \frac{d\theta}{dt} &= \frac{1}{1 + \left(\frac{400}{300}\right)^2} \cdot 20 \cdot \frac{1}{300} \\ &= \frac{20}{1 + 16/9} = 20 \cdot \frac{9}{25} \cdot \frac{1}{300} \end{aligned}$$

$$\frac{d\theta}{dt} = \frac{20}{300} \cdot \left(\frac{3}{5}\right)^2$$

$$= \frac{20 \cdot 9}{300 \cdot 25}$$

$$= \frac{60}{2500}$$

$$= \frac{6}{250}$$

$$= \frac{3}{125}$$

$$\boxed{\frac{3}{125}} \checkmark$$

rad/s



8. (8 points) Determine the value(s) of the absolute extreme values of  $f(t) = \frac{3t}{t^2+1}$  on the interval  $[-2, 0]$ .

$f$  is continuous on a closed interval: it must have an absolute maximum and minimum ✓

Find critical points: ✓

$$f'(t) = \frac{3(t^2+1) - 3t \cdot 2t}{(t^2+1)^2}$$

$$= \frac{3t^2 + 3 - 6t^2}{(t^2+1)^2}$$

$$= \frac{-3(t^2-1)}{(t^2+1)^2} \checkmark$$

$$t = -1 \checkmark \text{ and } t = 1 \text{ (crossed out)}$$

Evaluate at critical points and endpoints ✓

$$f(-2) = \frac{-6}{5} = -1.2 \checkmark$$

$$f(-1) = \frac{-3}{2} = -1.5 \checkmark$$

$$f(0) = 0 \checkmark$$

Absolute minimum value:

$-1.5 \checkmark$

Absolute maximum value:

$0 \checkmark$

9. (10 points) Which of the following functions satisfy the conditions of Rolle's Theorem on the interval  $[-1, 1]$ ?

$\times f(x) = 1 - x^{2/3}$  not differentiable at 0  
 $\checkmark g(x) = x^3 - 2x^2 - x + 2$   $g(-1) = g(1)$   
 $\times h(x) = \cos\left(\frac{\pi}{4}(x+1)\right)$   $h(-1) \neq h(1)$

Rolle's Theorem applies to:

- A. both  $f$  and  $g$   
B. both  $g$  and  $h$   
 C.  $g$  only  
D.  $h$  only  
E. All three

10. i. (3 points) Fill in the blanks to complete the statement of the Mean Value Theorem:

If  $f$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$ , then there is at least one point  $c$  in  $(a, b)$  such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- ii. (6 points) The  $f(x) = x + \frac{1}{x}$  on  $[1, 3]$  satisfies the conditions of the Mean Value Theorem. Find the point(s),  $c$ , that is/are guaranteed to exist by the Mean Value Theorem.

Derivative:  $f'(x) = 1 - \frac{1}{x^2}$

Slope of secant:  $\frac{f(3) - f(1)}{3 - 1} = \frac{3 + \frac{1}{3} - (1 + 1)}{2}$   
 $= \frac{1 + \frac{1}{3}}{2} = \frac{2}{3}$

$$f'(c) = 1 - \frac{1}{c^2} = \frac{2}{3}$$

$$\frac{1}{c^2} = \frac{1}{3}$$

$$c = \pm \sqrt{3}$$

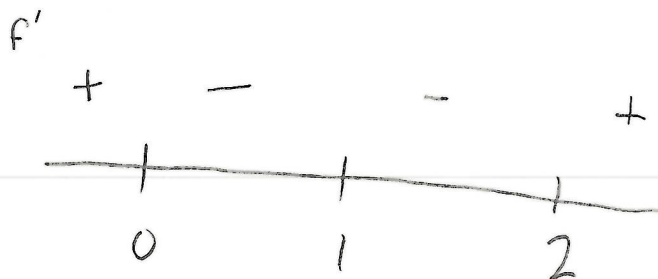
$\sqrt{3}$

11. Suppose that the derivative of a function  $h$  is given by:

$$h'(x) = x(x-1)^2(x-2)$$

i. (7 points) On what interval(s) is  $h$  increasing?

- A.  $(-\infty, 0)$
- B.  $(-\infty, 0)$  and  $(2, \infty)$
- C.  $(0, 2)$
- D.  $(0, 1)$  and  $(2, \infty)$
- E.  $(-\infty, 1)$



ii. (7 points) Where does  $h$  have a local maximum?

- A.  $x = 0$
- B.  $x = 2$
- C.  $x = 0$  and 1
- D.  $x = 0$  and 2
- E.  $x = 1$  and 2