Key

MA 16100 **Midterm Exam 3** **Summer 2019**

Name: _____

ID number: _____

Instructions:

1. This is a one-hour exam. If you continue to work on this exam after time is called it will be considered cheating and you will receive a zero.
2. There are 10 problems on this exam.
3. No books, notes, or calculators are allowed. Only a writing utensil, eraser, and water are allowed on your desk with the exam.
4. Turn off your cell phone.
5. Circle ***one and only one choice*** for each multiple-choice problem. Showing your work is NOT necessary for multiple-choice problems and no partial credit will be given for multiple-choice problems.
6. Legibly show all relevant work on non-multiple-choice problems. Partial credit will be given for steps leading to the correct solutions.
   Little or no work with a correct answer will receive little or no credit.

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

I agree to abide by the instructions above and have read and understood the above statement regarding academic integrity:

Signature: _____

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1. (10 points) Consider the following function:

$$h(x) = e^x(x-2)^2$$

i. (5 points) Using the Second Derivative Test, identify any local maxima or local minima of $h(x)$. If none exist, write NONE in the answer box.

$h'(x) = e^x(x-2)^2 + e^x \cdot 2(x-2)$ ✓

$= e^x(x^2 - 4x + 4 + 2x - 4)$

$= e^x(x^2 - 2x)$

$= e^x \cdot x(x-2)$ ✓

Critical points: $x = 0, x = 2$ ✓

-1 for not using 2D test

$h''(x) = e^x(x^2-2x) + e^x(2x-2)$

$= e^x(x^2-2)$ ✓

$h''(0) < 0$ ✓, $h''(2) > 0$ ✓

Local Maxima

$(0, 4)$ ✓

x-coordinate is ok

Local Minima

$(2, 0)$ ✓

ii. (5 points) Find any inflections points of $h(x)$. If none exist, write NONE.

$h''(x) = e^x(x^2-2)$ ✓

$x^2 - 2 = 0$ ✓

$x = \pm\sqrt{2}$

Inflection point(s)

$-\sqrt{2}, \sqrt{2}$ ✓ ✓

2 pts if they found where $h''(x) = 0$ even if the 2nd deriv is wrong.

2. (10 points) Let $f(x) = \dfrac{x^2+x+1}{x^2}$. Then $f'(x) = \dfrac{-x-2}{x^3}$ and $f''(x) = \dfrac{2(x+3)}{x^4}$.

i. (3 points) Does $f(x)$ have any symmetry? Explain why or why not.

$f(-x) = \dfrac{x^2-x+1}{x^2} \neq f(x)$ or $-f(x)$

No symmetry

ii. (2 points) Identify the horizontal and vertical asymptotes of f. If none, write NONE.

$\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to-\infty} f(x) = 1$

Horizontal asymptote(s)

$y = 1$

Vertical asymptote(s)

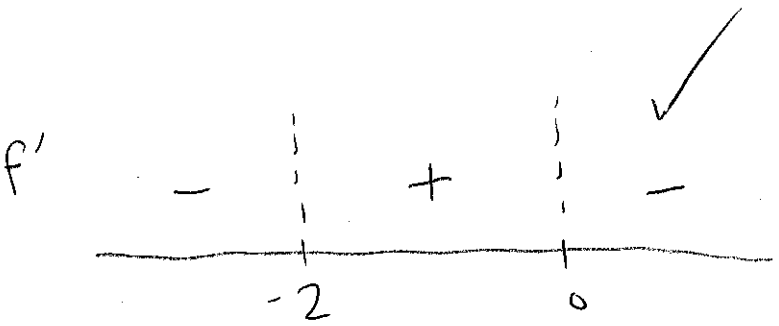
$x = 0$

THIS PROBLEM CONTINUED ON THE NEXT PAGE

Let $f(x) = \frac{x^2+x+1}{x^2}$. Then $f'(x) = \frac{-x-2}{x^3}$ and $f''(x) = \frac{2(x+3)}{x^4}$.

iii. (3 points) On what intervals is f increasing? Decreasing?

increasing: $f'(x)>0$ ✓, decreasing: $f'(x)<0$ ✓



Increasing on:

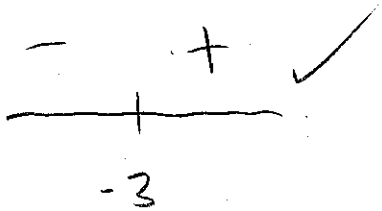
$(-2, 0)$ ✓

Decreasing on:

$(-\infty, -2), (0, \infty)$ ✓

iv. (2 points) On what intervals is $f(x)$ concave up? concave down?

concave up: $f''(x)>0$ ✓, concave down: $f''(x)<0$ ✓



Concave up on:

$(-3, \infty)$ ✓

$(-3,0), (0,\infty)$ technically the right answer!

Concave down on:

$(-\infty, -3)$ ✓

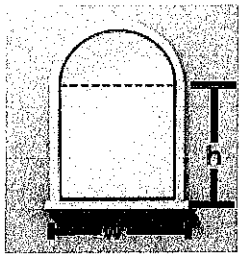
3. (10 points) What point on the line $y = 3x + 4$ is closest to the origin?

A. $(-4/3, 0)$
B. $(-1.2, 0.4)$
C. $(0, 4)$
D. $(1, 7)$
E. $(4, 0)$

4. (10 points) Use linear approximation to estimate the value of $\ln(1.07)$.

A. 0
B. 0.06
C. 0.07
D. 1
E. 1.07

5. (10 points) A window consists of a rectangular pane of glass surmounted by a semicircular pane of glass (see figure). If the perimeter of the window is 20 feet, determine the dimensions of the window which maximize the area of the window.



each

~~2 pts~~ 2 pts ✓ for misinterpretation

i. (4 points) Write down the **objective function** and **constraint(s)** in terms of the width (w) and height (h) of the window.

objective function: area ✓ $A = wh + \frac{1}{2}\pi\left(\frac{w}{2}\right)^2 = wh + \frac{\pi w^2}{8}$ ✓

constraint: perimeter ✓ $P = w + 2h + \frac{2\pi}{2}\left(\frac{w}{2}\right)$ ✓

$20 = w + 2h + \frac{w\pi}{2}$

ok if they already subbed

ii. (6 points) Determine the dimensions which maximize the area of the window.

full points

$2h = 20 - \left(1+\frac{\pi}{2}\right)w$

$h = 10 - \left(\frac{2+\pi}{4}\right)w$ ✓ solve for h or w

✓ write as function of 1 variable

$A(w) = w\left[10 - \left(\frac{2+\pi}{4}\right)w\right] + \frac{\pi}{8}w^2$

✓ take derivative

$A'(w) = 10 - \left(\frac{2+\pi}{4}\right)w - \left(\frac{2+\pi}{4}\right)w + \frac{\pi}{4}w$

✓ set to 0 to find crit pts.

$0 = 10 + \left(\frac{\pi}{4} - \frac{2+\pi}{2}\right)w$

✓ verify maximum

$\left(1 + \frac{\pi}{2} - \frac{\pi}{4}\right)w = 10$

$\left(1+\frac{\pi}{4}\right)w = 10$ ✓ find crit pt.

$w = \frac{40}{4+\pi}$ ✓

$h = 10 - \left(\frac{2+\pi}{4}\right)\left(\frac{40}{4+\pi}\right)$

$h = 10 - 10\left(\frac{2+\pi}{4+\pi}\right)$

$h = \frac{20}{4+\pi}$ ✓

6. (10 points) Evaluate the limit

$$\lim_{x\to 1^+} x^{1/(1-x)}$$

A. 0

B. $1/e$

C. 1

D. e

E. ∞

$$L = \lim_{x\to 1^+} \frac{\ln x}{1-x} = \lim_{x\to 1^+} \frac{\frac{1}{x}}{-1} = -1$$

$$e^L = e^{-1}$$

7. (10 points) Evaluate the following indefinite integral: $\int \frac{(5s+3)^2}{s} ds$.

$$= \int \frac{25s^2 + 30s + 9}{s} \, ds$$

$$= \int 25s + 30 + \frac{9}{s} \, ds$$

$$= \frac{25}{2} s^2 + 30s + 9 \ln|s| + C$$

$$\boxed{\frac{25}{2} s^2 + 30s + 9 \ln|s| + C}$$

8. (10 points) Evaluate the following indefinite integral: $\int \left(\frac{4}{1+x^2} - 2 \right) dx$

$$= \int \frac{4}{1+x^2} \, dx - \int 2 \, dx$$

$$= 4 \int \frac{dx}{1+x^2} - 2 \int dx$$

$$= 4 \tan^{-1} x - 2x + C$$

$$\boxed{4 \tan^{-1}(x) - 2x + C}$$

9. (10 points) The acceleration of an object moving along a line is given by $a(t) = 6t$ m/s^2 with an initial velocity of $v(0) = 3$ m/s, and an initial position of $s(0) = 4$ m. Find the position of the object after 2 seconds (i.e. find $s(2)$).

A. 6 m

B. 12 m

C. 14 m

D. 18 m

E. 24 m

10. (10 points) Compute the right Riemann sum of $g(t) = t^2 - 1$ on $[2, 5]$ with 3 subintervals.

$\Delta t = \frac{5-2}{3} = 1$

$t_0 = 2,\ t_1 = 3,\ t_2 = 4,\ t_3 = 5$

$g(t_1^*) = g(3) = 8$

$g(t_2^*) = g(4) = 15$

$g(t_3^*) = g(5) = 24$

$g(t_0^*)\Delta t + g(t_1^*)\Delta t + g(t_2^*)\Delta t$

$= 8 \cdot 1 + 15 \cdot 1 + 24 \cdot 1$

$= 47$

47.

You may take this page with you after the exam

Question #	Point value	Your answer	Correct answer
1	10		
2	10		
3	10		
4	10		
5	10		
6	10		
7	10		
8	10		
9	10		
10	10		

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