1. The length of a rectangle is measured at 30cm and the width of a rectangle is measured at 24cm with an error in measurement of at most 0.1cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

Solution

The area of the rectangle is given by $A = lw$ where $l$ is the length and $w$ is the width. We can write $A(l, w) = lw$ and estimate that $\Delta A \approx dA = \frac{\partial A}{\partial l} \Delta l + \frac{\partial A}{\partial w} \Delta w$.

Now $\frac{\partial A}{\partial l} = w$ and $\frac{\partial A}{\partial w} = l$ so we can estimate that the maximum error is $\Delta A \approx w \Delta l + l \Delta w = (24)(\pm 0.1) + (30)(\pm 0.1) = 5.4\text{cm}^2$, by making the signs of each term match up.