# MA 16010 - Quiz 12 Solution - Fall 2017

**Directions**: Show all the work leading to your answers. Some correct work with an incorrect answer will earn you partial credit. A correct answer without justification will not earn you full credit.

1. Find the point on the curve $y = x - 2$ closest to the point $(0, 5)$. Justify your answer.

   **Solution:**

   The objective function is $d = \sqrt{(x-0)^2 + (y-5)^2}$ and the constraint is that $y = x - 2$. Plugging the constraint into the objective function gives:

$$d(x) = \sqrt{x^2 + (x-2-5)^2}$$
$$d(x) = \sqrt{x^2 + (x-7)^2}$$
$$d(x) = \sqrt{x^2 + x^2 - 14x + 49}$$

   We can minimize the function $D(x) = [d(x)]^2 = 2x^2 - 14x + 49$ instead.

   Finding critical numbers: $D'(x) = 4x - 14$. The only critical number is $x = \frac{7}{2}$. To check that this gives a relative minimum, we can use the Second Derivative Test: $D''(x) = 4 > 0$ so, in particular, $D''\left(\frac{7}{2}\right) > 0$ which means that $D\left(\frac{7}{2}\right)$ is a relative minimum and since $x = \frac{7}{2}$ is the only critical number, it must be that $D\left(\frac{7}{2}\right)$ is the absolute minimum.

   The point on the curve $y = x - 2$ closest to $(0, 5)$ is the point $\left(\frac{7}{2}, \frac{3}{2}\right)$