1. Find the area of the region bounded by the curves
   \[ x = 2 - y^2 \text{ and } y = -2x + 1 \]

   (a) (3 points) Sketch the curves on a single graph and shade in the region bounded by the curves
   (b) (1 point) Rewrite the curves so that they are functions of y
   (c) (1 point) Indicate which curve is the rightmost and which curve is the leftmost in the region
   (d) (2 points) Find the points at which the two curves intersect
   (e) (3 points) Set up and evaluate the integral to compute the area of the region

Solution

1.

(a) Your sketch should look something like this:
(b) Solve each curve for $x$ as a function of $y$: $x = 2 - y^2$ is done already, $y = -2x + 1$ is the same thing as $x = \frac{1}{2}(y - 1)$

(c) $x = 2 - y^2$ is the rightmost and $x = \frac{-1}{2}(y - 1)$ is the leftmost

(d) To find the points of intersection, set the two equations for $x$ equal to each other and solve for $y$:

\[
\begin{align*}
2 - y^2 &= \frac{-1}{2}(y - 1) \\
4 - 2y^2 &= -y + 1 \\
2y^2 - y - 3 &= 0
\end{align*}
\]

use the quadratic formula or attempt to factor the polynomial:

\[
y = \frac{1 \pm \sqrt{1 - 4(-3)(2)}}{4} = \frac{1 \pm \sqrt{25}}{4} = \frac{1 \pm 5}{4}
\]

so the two points are $y = \frac{3}{2}$ and $y = -1$

(e) The integral we must compute is: $\int_{-1}^{2} (2 - y^2) - \frac{-1}{2}(y - 1) dy = \int_{-1}^{2} 2 - y^2 + \frac{1}{2}y - \frac{1}{2} dy = 2y - \frac{1}{3}y^3 + \frac{1}{4}y^2 - \frac{1}{2}y|_{-1}^{2} = (3 - \frac{1}{3}\frac{4}{3} + \frac{1}{4}\frac{4}{4} - \frac{1}{2}\frac{2}{2}) - (-2 + \frac{1}{3} + \frac{1}{2} + \frac{1}{2}) = 3 - \frac{9}{8} + \frac{9}{16} - \frac{3}{4} + 2 - \frac{1}{3} - \frac{1}{4} - \frac{1}{2} = \frac{144 - 54 + 27 - 36 + 96 - 16 - 12 - 24}{48} = \frac{125}{48}$