Problem Set 4

Due October 11th at 4 pm in room 2-285.

Hand in parts 1 and 2 separately. Put your name on each part.

Part 1

1. Problem 1 from page 78.
2. Let X be a complete metric space with metric d, and let $f\colon X \to X$ be a *contraction*, meaning that there exists $\lambda < 1$ such that

$$d(f(x), f(y)) \leq \lambda d(x, y)$$

for all $x, y \in X$. Prove that there is a unique point $x_0 \in X$ such that $f(x_0) = x_0$.

Part 2

3. (a) Show that if $(a_n)_{n\in\mathbb{N}}$ is a convergent sequence of nonnegative real numbers then $(\sqrt{a_n})_{n\in\mathbb{N}}$ is also convergent and

$$\lim_{n\to\infty} \sqrt{a_n} = \sqrt{\lim_{n\to\infty} a_n}$$

   (b) Problem 2 from page 78
4. Let K be a compact metric space, and $\{G_\alpha\}_{\alpha\in A}$ an open cover of K. Prove that there exists $\varepsilon > 0$ such that for every $x \in K$ there exists $\alpha \in A$ such that $N_\varepsilon(x) \subset G_\alpha$.