

## Problem Set 7

Due November 1st at 4 pm in room 2-285.

Hand in parts 1 and 2 separately. Put your name on each part.

### Part 1

1. Problem 14 from page 100.
2. Let  $K$  be a compact metric space with metric  $d$  and suppose  $f: K \rightarrow K$  obeys  $d(f(x), f(y)) \leq d(x, y)$  for all  $x, y \in K$ . Prove that the following are equivalent:
  - (a)  $f$  is surjective.
  - (b)  $d(f(x), f(y)) = d(x, y)$  for all  $x, y \in K$ .

### Part 2

3. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be continuous and suppose that  $\lim_{x \rightarrow +\infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$  both exist and are finite. Prove that  $f$  is uniformly continuous.
4. Problem 1 from page 114.
5. Problem 6 from page 114.