18.104
Spring 2013

Homework 1

Due February 12th by the beginning of class.

Problem: Let $u\colon C_c^\infty((0,\infty)) \to \mathbb{C}$ be defined by
$$\langle u, \phi\rangle = \int_0^\infty e^{1/x}\phi(x)dx.$$

(1) Prove that $u \in \mathscr{D}'((0,\infty))$
(2) Prove that there is no $v \in \mathscr{D}'(\mathbb{R})$ whose restriction to $(0,\infty)$ is u.

Hint: The first part has a very short solution. For the second, show that the estimate [FrJo, (1.3.1)] in the definition of a distribution leads to a contradiction by considering a suitable family of test functions ϕ_ϵ, parametrized by $\epsilon \in (0,1)$. One possible choice is $\phi_\epsilon(x) = \phi(x/\epsilon)$, for some $\phi \in C_c^\infty((0,\infty))$ nonnegative and not identically zero.

LaTeX

Solution:

References

[FrJo] G. Friedlander and M. Joshi. The Theory of Distributions, second edition, Cambridge University Press, 1998.