Homework 11

Due April 9th by the beginning of class.

Problem: Prove that if $u \in \mathscr{S}'(\mathbb{R}^n)$ and $f \in \mathscr{S}(\mathbb{R}^n)$ then $u * f$ can be defined as in the remark following [FrJo, Theorem 8.4.2]. Show that given $u \in \mathscr{S}'(\mathbb{R}^n)$, there is $f \in \mathscr{S}(\mathbb{R}^n)$ such that $u * f = u$ if and only if $\hat{u} \in \mathscr{E}'(\mathbb{R}^n)$. (This shows that regularization theorems like [FrJo, Theorem 5.2.1] are trickier when both the function and distribution have noncompact support.)

Hint: Solving $u * f = u$ is equivalent to solving $\hat{u}\hat{f} = \hat{u}$. Use $\lim_{|\xi| \to \infty} \hat{f}(\xi) = 0$ to prove one implication, and make $\hat{f} = 1$ near the support of $\hat{u}$ for the other.

Solution:

References

[FrJo] G. Friedlander and M. Joshi. The Theory of Distributions, second edition, Cambridge University Press, 1998.