Homework 3

Due February 21st by the beginning of class.

Problem: Calculate the limits as $n \to \infty$ of the following sequences of distributions on $\mathbb{R}$:

$$u_n = \frac{n}{1 + n^2 x^2}, \quad v_n = \frac{n^3 x}{(1 + n^2 x^2)^2}, \quad w_n = \frac{n^3 \sin x}{(1 + n^2 x^2)^2}.$$

Hint: Reduce each limit to the previous one. You may use without proof the dominated convergence theorem, which says that if $f_n(x) \to f(x)$ for every $x \in \mathbb{R}$, and if $|f_n(x)| \le g(x)$ for every $n \in \mathbb{N}$ and $x \in \mathbb{R}$ for some function g with $\int_{\mathbb{R}} g < \infty$, then $\lim_{n \to \infty} \int_{\mathbb{R}} f_n = \int_{\mathbb{R}} f$.

Solution:

REFERENCES

[FrJo] G. Friedlander and M. Joshi. The Theory of Distributions, second edition, Cambridge University Press, 1998.