18.104
Spring 2013

## Homework 4

Due February 26th by the beginning of class.

**Problem:** Prove that
$$\lim_{n\to\infty} \frac{n^2 x}{1+n^2x^2} = \text{p.v.}\frac{1}{x}$$
in $\mathscr{D}'(\mathbb{R})$, where $\text{p.v.}\frac{1}{x}$ denotes the principal value distribution as in [FrJo, page 19].

*Hint:* Use the fact that
$$\langle \text{p.v.}\frac{1}{x}, \phi\rangle = \int_{-1}^{1} \frac{\phi(x)-\phi(0)}{x}dx + \int_{\mathbb{R}\setminus[-1,1]} \frac{\phi(x)}{x}dx.$$
You may use without proof the dominated convergence theorem, which says that if $f_n(x) \to f(x)$ for every $x \in \mathbb{R}$, and if $|f_n(x)| \le g(x)$ for every $n \in \mathbb{N}$ and $x \in \mathbb{R}$ for some function $g$ with $\int_{\mathbb{R}} g < \infty$, then $\lim_{n\to\infty} \int_{\mathbb{R}} f_n = \int_{\mathbb{R}} f$.

*Not required:* What about $\lim \frac{n^2 f(x)}{1+n^2x^2}$, where $f \in C^\infty(\mathbb{R})$?

**Solution:**

### References


[FrJo] G. Friedlander and M. Joshi. The Theory of Distributions, second edition, Cambridge University Press, 1998.