18.104
Spring 2013

Homework 6

Due March 5th by the beginning of class.

Problem: Prove that $u \in \mathscr{D}'(\mathbb{R}^2)$ solves the system

$$(x_1^2 - x_2^2)u = 0, \qquad x_1 x_2 u = 0, \tag{1}$$

if and only if there exist $c_0, c_1, c_2, c_3 \in \mathbb{C}$ such that

$$u = c_0\delta + c_1\partial_1\delta + c_2\partial_2\delta + c_3(\partial_1^2 + \partial_2^2)\delta.$$

Hint: Deduce $x_1^3 u = 0$ and $x_2^3 u = 0$ from (1), and show these imply $u = \sum_{|\alpha|\le N} c_\alpha \partial^\alpha \delta$ for some $N \in \mathbb{N}$. The additional restrictions on the c_α follow from these two equations, as well as the individual equations in (1) (here it may be helpful to use the formulas for the coefficients c_α provided at the end of the proof of Theorem 3.2.1 in [FrJo]).

Solution:

References

[FrJo] G. Friedlander and M. Joshi. The Theory of Distributions, second edition, Cambridge University Press, 1998.