18.104
Spring 2013

Homework 8

Due March 12th by the beginning of class.

Problem: Prove that $u \in \mathscr{D}'(\mathbb{R}^n)$ solves $x_n u = 0$ if and only if $u = v(x') \otimes \delta(x_n)$ for some $v \in \mathscr{D}'(\mathbb{R}^{n-1})$. (Notation is as in [FrJo, Theorem 4.3.4].)

Hint: Fix $\chi \in C_c^\infty(\mathbb{R})$ with $\chi(0) = 1$. Show that for any $\phi \in C_c^\infty(\mathbb{R}^n)$ there is $\tilde{\phi} \in C_c^\infty(\mathbb{R}^n)$ such that $\phi(x) = \phi(x', 0)\chi(x_n) + x_n\tilde{\phi}(x)$ (this is similar to the $m = 1$ case of [FrJo, (2.7.7)]). Proceed as in the proof of [FrJo, Theorem 4.3.4].

Solution:

References

[FrJo] G. Friedlander and M. Joshi. The Theory of Distributions, second edition, Cambridge University Press, 1998.