MA 341 final review problems
Version as of December 8th.

The final exam will be in class on Saturday, December 18th from 8 am to 10 am in SMTH 118. Justify your answers. No notes or electronic devices allowed. Most of the problems on the exam will be closely based on problems, or on parts of problems, from the list below. Some of them reference the previous exam reviews `https://www.math.purdue.edu/~kdatchev/341/mid1prac.pdf` and `https://www.math.purdue.edu/~kdatchev/341/mid2prac.pdf` as well as the extra examples `https://www.math.purdue.edu/~kdatchev/341/examples.pdf`. Please let me know if you have a question or find a mistake.

1. Evaluate
$$\frac{d}{dx}\int_x^{x^2}\sin(t^2)dt.$$

2. Let $f\colon \mathbb{R}\to\mathbb{R}$ be a differentiable function. Prove that if f' is bounded, then f is uniformly continuous.

3. For which values of k is $f(x)=x^k$ uniformly continuous on $[1,\infty)$? If it is uniformly continuous, given any $\varepsilon>0$ find $\delta>0$ such that $|f(x)-f(y)|<\varepsilon$ when $|x-y|<\delta$. If it is not, find $\varepsilon>0$ and sequences x_n and y_n in $[1,\infty)$ such that $|x_n-y_n|<1/n$ and $|f(x_n)-f(y_n)|\geq\varepsilon$.

4. Let b_n be a bounded sequence, and let $a_n=6n^{3/2}+b_n$. For each $p>0$, evaluate
$$\lim_{n\to\infty}\frac{a_n}{n^p}.$$

5. For each of the following functions, determine the pointwise limit $f(x)$ on the indicated interval, and decide whether the convergence is uniform. If the convergence is uniform, find a sequence of real numbers $B_n\to 0$ such that $|f_n(x)-f(x)|\leq B_n$ for all x in the interval. If it isn't, find $\varepsilon>0$ and a sequence x_n such that $|f_n(x_n)-f(x_n)|\geq\varepsilon$ for all n.
   (a) $f_n(x)=n^{-1}\sin x$ on $\mathbb{R}$.
   (b) $f_n(x)=e^{nx}$ on $(-\infty,-1)$.
   (c) $f_n(x)=e^{-nx^2}$ on $[-10,10]$.
   (d) $f_n(x)=nx/(1+n+x)$ on $[0,10]$.
   (e) $f_n(x)=nx/(1+n+x)$ on $[0,\infty)$.

6. Problems 1, 2, and 6 from the first midterm review. See also the extra examples for Sections 2.5 and 7.4.

7. Problems 3, 4, and 5 from the second midterm review. See also the extra examples for Sections 11.1, 13.2 and 13.3.