

Homework 3

Due September 15th at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake.

1. Exercise 4.2.1 from page 58. Use the fact that

$$\int_0^b (1+u^2)^{-1} du = \arctan(b).$$

2. Exercise 4.2.2 from page 58. Use the fact that

$$\int_0^b (1-u^2)^{-1} du = \frac{1}{2} \ln \left(\frac{1+b}{1-b} \right).$$

3. The previous result gives a more efficient way of calculating $\ln 2$ than Example 4.2. Calculate the first 4 terms of the sequence using a calculator. What approximations of $\ln 2$ do they each correspond to? Write the approximations as fractions and as decimal expansions. Comparing them to the sequence of digits in $\ln 2$ given here <https://oeis.org/A002162>, how many digits after the decimal are correct in each case?
4. Evaluate the following limits by using the Linearity, Product, Quotient, and Squeeze Theorems (Theorems 5.1 and 5.2) to reduce them to limits studied previously.

(a)

$$\lim_{n \rightarrow \infty} \frac{2^n + \cos n}{2^n + \sin n}$$

(b)

$$\lim_{n \rightarrow \infty} \frac{3n^2 5^n + 3n^7}{2^n + 6n^2 5^n}$$