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MA 341
Fall 2021

Homework 6

Due Wednesday, October 20th at the beginning of class. Note that we are back to having homework due on Wednesdays. Justify your answers. Please let me know if you have a question or find a mistake.

1. Find a positive real number b and a real number c such that the number of real solutions to

$$\cos x = \frac{x^2}{A}$$

is at least $c + b\sqrt{A}$, for any $A > 0$.

2. Let $f\colon [0,\infty) \to \mathbb{R}$ be a continuous function such that $f(x) > 0$ for all $x \geq 0$ and such that

$$\lim_{x\to\infty} f(x) = 0.$$

(a) Prove that f is bounded on $[0,\infty)$.

Hint: Consider separately the intervals $[0,N]$ and (N,∞), where N is a strategically chosen large number.

(b) Prove that f has a maximum on $[0,\infty)$.

Hint: Use the same approach as in part (b), but explain how to choose N in such a way that the maximum is attained on $[0,N]$.

(c) Prove that f has no minimum on $[0,\infty)$.

Hint: For any $a \geq 0$, prove that there is x such that $f(x) < f(a)$.