

Homework 9

Due Wednesday, December 1st at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake.

1. Exercise 22.2.1 from page 322.
2. Exercise 22.1.3 from page 322. Instead of working directly with the definition, you may use the result of Exercise 22.2.1 above.
3. For each of the following functions, determine the pointwise limit $f(x)$ on the indicated interval, and decide whether the convergence is uniform. If the convergence is uniform, find a sequence of real numbers $B_n \rightarrow 0$ such that $|f_n(x) - f(x)| \leq B_n$ for all x in the interval. If it isn't, find $\varepsilon > 0$ and a sequence x_n such that $|f_n(x_n) - f(x_n)| \geq \varepsilon$ for all n .
 - (a) $f_n(x) = x^{1/n}$ on $[0, 1]$.
 - (b) $f_n(x) = n^{-1}e^{-x^2}$ on $\mathbb{R}$.
 - (c) $f_n(x) = x^n - x^{2n}$ on $[0, 1]$.

Hint: For part (c), for each n , find the maximum of f_n .

4. Exercise 22.2.2(d) from page 322.
5. Exercise 22.4.4 from page 324.

Hint: Estimate the fraction in a different way depending on whether $x \leq 1$ or $x \geq 1$.

6. Problem 22-1 from page 325.

Hint: Use the series for $\ln(1+x)$ from page 315.