

MA 416 first midterm review problems

Version as of September 26th.

The first midterm will be in class on Tuesday, September 30th. It covers up through Section 4.5. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems from the list below. Justify your answers, and do simple simplifications but not complicated ones. Please let me know if you have a question or find a mistake.

1. If we roll 10 dice, what are the chances that exactly 8 of them match and the other two are singletons?
2. If we deal out 20 cards from a deck, what are the chances at least one of them is an ace or a spade?
3. A metal detector detects metal correctly $1/4$ of the time, and nonmetal $4/5$. Suppose $2/3$ of the material received by the detector has metal. If a given material is detected as having metal, what are the chances it actually has metal?
4. Find the expected value and variance of the number of 2s that show up when three dice are rolled. What is the associated probability mass function?
5. If A and B each flip n fair coins, what is the probability that they each flip the same number of heads, for $n = 1$, for $n = 2$, and for general n ? *Hint:* Use the identity $\binom{2m}{m} = \sum_{k=0}^m \binom{m}{k}^2$.

Also the following from the homework / self-test:

- Chapter 1, Problems 10, 34.
- Chapter 2, Problems 10, 18, 20, 25, 34, 37, 54, Theoretical Exercise 6.
- Chapter 3, Theoretical Exercise 2, Self-Test 21, 34, 36, 37.
- Chapter 4, Self-Test 2, 3, 5.

Consider also variants of problems: for example, many problems can be converted between versions about coins, dice, and cards, either fair or biased.

Here are short answers; a proper solution includes some clear steps leading to these answers. Note also the Self-Test problems have complete solutions at the back of the book.

1. $25/6^7$
2. $1 - \frac{32 \cdot 31 \cdots 17}{52 \cdot 51 \cdots 37}$
3. $5/7$
4. $p(0) = 125/216, p(1) = 75/216, p(2) = 15/216, p(3) = 1/216, EV = 1/2, Var = 5/12.$
5. $1/2, 3/8,$ and $\binom{2n}{n}/4^n$. (You may enjoy comparing these values to $1/\sqrt{\pi n}$; this is *Stirling's approximation*.)

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