

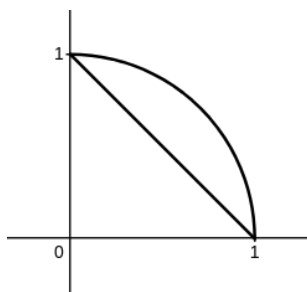
MA 416 second midterm review problems

Version as of November 2nd.

The second midterm will be in class on Tuesday, November 4th. It covers up through Chapter 5. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems from the list below. Justify your answers, and do simple simplifications but not complicated ones. Please let me know if you have a question or find a mistake.

1. A certain medical test comes back positive 90% of the time if the patient has the disease, and 20% of the time otherwise. If 1% of the population has the disease, what is the probability that a patient who tests positive has the disease?
2. Consider a population in which 10% of people are left-handed. Give answers to the following both as an exact number using binomial coefficients and in terms of the cumulative standard normal distribution function Φ using the table on the next page.
 - (a) What are the chances that in a random group of 100 people exactly 10 will be left-handed?
 - (b) What are the chances that in a random group of 100 people at least 5 will be left-handed?
 - (c) What is the most likely number of left-handed people in a group of 49? What are the chances of having exactly that number of left-handed people in such a group?
3. Let Y be the y -coordinate of a point chosen uniformly along the perimeter of the region below (bounded by $x^2 + y^2 = 1$ and $x + y = 1$). Find the cumulative distribution function, probability density function, mean, and variance of Y . You may use the formulas

$$\frac{1}{\sqrt{1-y^2}} = \frac{d}{dy} \arcsin y, \quad \frac{-y}{\sqrt{1-y^2}} = \frac{d}{dy} \sqrt{1-y^2}, \quad \frac{y^2}{\sqrt{1-y^2}} = \frac{d}{dy} \frac{\arcsin y - y\sqrt{1-y^2}}{2},$$



Also the following from the homework / self-test:

- Chapter 3, Problems 60, 89, Theoretical Exercise 11.
- Chapter 4, Problem 39, Theoretical Exercise 4, Self-Test 9, 11, 15.
- Chapter 5, Problems 4a, 7, 11, Self-Test 4, 8.

Use the table for the cumulative standard normal distribution on the next page.

Table 5.1 Area $\Phi(x)$ Under the Standard Normal Curve to the Left of X .

[illegible]

Here are short answers; a proper solution includes some clear steps leading to these answers. Note also the Self-Test problems have complete solutions at the back of the book.

1. $1/23$.

2. (a) $\binom{100}{10} \frac{9^{90}}{10^{100}}$, $2\Phi(1/6) - 1 \approx 2 * 0.5675 - 1 \approx 0.13 = 13\%$.

(b) $1 - 10^{-100} \sum_{j=0}^4 \binom{100}{j} 9^{100-j}$, $\Phi(11/6) \approx 0.9671 \approx 97\%$.

(c) $5, \binom{49}{5} \frac{9^{44}}{10^{49}}$, $\Phi(6/21) + \Phi(4/21) - 1 \approx \Phi(0.29) + \Phi(0.19) - 1 \approx 0.6141 + 0.5753 - 1 = 0.1894 \approx 19\%$.

3.

$$F(y) = \begin{cases} 0, & y \leq 0, \\ \frac{\sqrt{2}y + \arcsin y}{\sqrt{2} + \frac{\pi}{2}}, & 0 < y < 1, \\ 1, & 1 \leq y. \end{cases}$$

$$f(y) = \begin{cases} \frac{\sqrt{2}}{\sqrt{2} + \frac{\pi}{2}} + \frac{1}{(\sqrt{2} + \frac{\pi}{2})\sqrt{1-y^2}}, & 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Mean is $\frac{2+\sqrt{2}}{2\sqrt{2}+\pi}$, variance is $\frac{2}{2\sqrt{2}+\pi} \left(\frac{\sqrt{2}}{3} + \frac{\pi}{4} - \frac{3+2\sqrt{2}}{2\sqrt{2}+\pi} \right)$.