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MA 442
Spring 2021

Homework 4

Due February 24th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake.

1. Use the results of Section 1-2 of Seeley's book to find $u(r, \theta)$, satisfying (1-2), (1-3), (1-4), and (1-5), with
   (a) $f(\theta) = 1$,
   (b) $f(\theta) = \sin\theta$,
   (c) $f(\theta) = \sin^2\theta = \frac{1-\cos 2\theta}{2}$.

   Write your final answers without any complex numbers. Note that for these problems infinite sums are not needed.

2. Rewrite the answers from the last problem in terms of x and y using $x = r\cos\theta$ and $y = r\sin\theta$. Recall that $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

3. Let $B > 0$ be given, and let A_n be a sequence of complex numbers such that $|A_n| \leq B$ for all integers n, and let
$$u(r, \theta) = \sum_{-\infty}^{\infty} A_n r^{|n|} e^{in\theta},$$
   for all $r \in [0, 1)$ and $\theta \in \mathbb{R}$.
   (a) Show that A_0 is the average value of u in the sense that
$$A_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} u(r, \theta) d\theta, \quad \text{for any } r \in (0, 1),$$
   by integrating both sides of the formula for u.
   (b) Give a bound for the rate at which u converges to A_0 at $r = 0$ by finding a continuous function $g\colon [0, 1) \to \mathbb{R}$, written in terms of B without using any series, such that $g(0) = 0$ and
$$|u(r, \theta) - A_0| \leq g(r),$$
   for all $r \in [0, 1)$ and $\theta \in \mathbb{R}$.

4. Exercise 1-4 from Seeley page 14. Don't use a calculator and omit finding $u(\frac{1}{2}, \pi)$ to two decimals. You may follow the answer in the back of the book, but if you do then carefully justify each step. The integral test mentioned there is the statement that if

f is a positive and decreasing function, then

$$\sum_{k=K+1}^{\infty} f(k) \leq \int_{K}^{\infty} f(x)dx.$$

5. Exercise 1-5 from Seeley page 14. Note there is a typo in the statement and (1-8) should be (1-9).