

Homework 8

Due March 24th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake.

- Find the measure of the set of $x \in \mathbb{R}$ such that $\sin x = 1/2$.

Hint: Write the points of the set in a sequence, or in a few sequences.

- Let $p \in \mathbb{R}$ be given and let

$$f_n(x) = \begin{cases} n^p, & \text{when } |x| \leq n^{-1}, \\ 0, & \text{otherwise.} \end{cases}$$

For which values of p do we have each of the following?

(a)

$$\lim_{n \rightarrow \infty} f_n(x) = 0 \text{ for almost every } x.$$

(b)

$$\lim_{n \rightarrow \infty} \sup_{x \in \mathbb{R}} |f_n(x)| = 0.$$

(c)

$$\lim_{n \rightarrow \infty} \int |f_n| = 0.$$

(d)

$$\lim_{n \rightarrow \infty} \int |f_n|^2 = 0.$$

- Let

$$f(\theta) = \begin{cases} -1, & \text{when } -\pi \leq \theta \leq \pi/2, \\ 1, & \text{when } \pi/2 \leq \theta < \pi \\ 2, & \text{when } \theta = \pi, \end{cases}$$

and extend f to be 2π -periodic. Sketch the graph of f , and find $\int_0^{\pi/4} f$, $\int_0^{17\pi/4} f$, and $\int_0^{2021\pi/4} f$.

- Exercise 4.9 of <https://www.math.purdue.edu/~kdatchev/428/ffa.pdf>.