MA 442 final review problems

Version as of April 25th.

The final will be on Tuesday, April 30th from 1 to 3 pm in BRNG 1245. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Justify your answers. Please let me know if you have a question or find a mistake. There are some hints on the last page.

1. Problems 1, 5, 6, 7 from the midterm review `https://www.math.purdue.edu/~kdatchev/442/midprac.pdf`.

2. Shifrin 8.2.11 parts (b) and (e), the corresponding parts of 8.2.12, and 8.5.6.

3. Let $U \subset \mathbb{R}^n$ and $V \subset \mathbb{R}^m$ be nonempty open sets. State and use a fact from linear algebra to show that if $f\colon U \to V$ is invertible, with f and f^{-1} both differentiable, then $m = n$.

4. Let M be a sphere in $\mathbb{R}^3$ which encloses a ball of volume 5. Let M be oriented so that $\int_M x dy \wedge dz > 0$. Use Stokes' theorem to evaluate
$$\int_M e^z dy \wedge dz + (\cos x + 2y) dz \wedge dx + (3x^2 - 4y^2) dx \wedge dy.$$

5. Let $a > 0$ be given, and let M be the torus in $\mathbb{R}^4$ given by the equations
$$w^2 + x^2 = y^2 + z^2 = a^2.$$
Find the surface area of the torus. Find the lengths of the six shortest geodesics on M from $(w, x, y, z) = (a, 0, a, 0)$ to $(w, x, y, z) = (a, 0, 0, a)$. Show that, for any real number N, there are non-self-intersecting geodesics on M from $(w, x, y, z) = (a, 0, a, 0)$ to $(w, x, y, z) = (a, 0, 0, a)$ of length greater than N and find the length of one such geodesic.

6. Let A be an $m \times n$ matrix, and let $f\colon \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}$ be given by $f(x, y) = x \cdot (Ay)$. Given $v = (a, b)$ and $w = (x, y)$ in $\mathbb{R}^m \times \mathbb{R}^n$, find $\frac{d}{dt}\big|_{t=0} f(v + tw)$.

For geodesics, use the following fact, which is a consequence of uniqueness of solutions to the geodesic equation: in coordinates in which the metric tensor is Euclidean, geodesics are given by straight lines parametrized at constant speed.

8.2.11(b) is $18\,dv$ and (e) is $\sin^2 u\,du \wedge dv$. Taking d of those gives 0. 8.5.6 is πa^4.