Kiril Datchev MA 442 Spring 2024

Homework 1

Due January 18th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. There are some hints on the second page.

- 1. Shifrin 3.1.4, 3.1.13, 3.2.7, 3.2.14a, 3.5.5, 3.5.7d.
- 2. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} 1, & 0 < |y| < x^2, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Sketch the set of points (x, y) in \mathbb{R}^2 such that f(x, y) = 1.
- (b) Show that f is not continuous at (0,0).
- (c) Show that $\lim_{t\to 0} f(at, bt) = 0$ and that $D_{(a,b)}f(0,0) = 0$ for any real a and b by finding an explicit interval I in terms of a and b, as large as possible, such that f(at, bt) = 0 when $t \in I$.

Hints:

3.1.13. If f(a + tv) = f(a) + tq + r(t), where $r(t)/t \to 0$ as $t \to 0$, then $D_v f = q$.

3.2.14a. Use a similar approach to 3.1.13.

3.5.5. First compute the derivative of $|g(t)|^2 = g(t) \cdot g(t)$, then use that and the chain rule to compute the derivative of $|g(t)| = (|g(t)|^2)^{1/2}$, and then follow the hint in the book.

3.5.7d. Use $2\cos^2(t/2) = 1 + \cos t$. Also, this is not required and won't be graded, but if you're feeling lively look at 3.5.14 and try to check the conclusion in the footnote of that problem; this discovery by Huygens four hundred years ago was an important advance in the problem of keeping accurate time.

2(b). Find a sequence of points $p_n = (x_n, y_n)$ such that $\lim f(p_n) \neq f(\lim p_n)$.