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MA 442
Spring 2021

Homework 10

Due April 21st on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake.

1. Evaluate
$$\int_C \sin z dx + \cos(\sqrt{y})dy + x^3 dz,$$
where C is the line segment from $(1,0,0)$ to $(0,0,3)$.

2. Let $\alpha = yzdx + xzdy + xydz$.
   (a) Find a function f such that $df = \alpha$.
   (b) Evaluate $\int_C \alpha$, where C is the parametric curve $(\cos t, e^t, \ln t)$, $1 \le t \le 2$.

3. Let $\varphi(u,v) = (x(u,v), y(u,v), z(u,v)) = (uv, u+v, u^v)$. Let $\alpha = xdy - ydx$ and $\beta = dx \wedge dy + x^2 dy \wedge dz$. Find $\varphi^*\alpha$, $\varphi^*\beta$, and $\varphi^*(\alpha \wedge \beta)$, when $u > 0$ and $v > 0$.

4. Let (w,x,y,z) be the standard coordinates on $\mathbb{R}^4$. Let
$$\alpha = dw \wedge dx + dy \wedge dz, \qquad \beta = dw \wedge dy + dx \wedge dz.$$
Let a and b be given positive numbers, and let M be the surface given by
$$w^2 + x^2 = a^2, \qquad y^2 + z^2 = b^2, \quad w > 0, \quad x > 0, \quad y > 0, \quad z > 0.$$
   (a) Simplify $\alpha \wedge \alpha$ and $\alpha \wedge \beta$.
   (b) Show that the differential form β is nowhere vanishing on M by pulling it back with a coordinate chart φ.
   (c) Evaluate $\int_M \alpha$ and $\int_M \beta$, where M is oriented so that β is positive.