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Homework 2

Due January 25th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. There are some hints on the second page.

- Look at Theorem 2.4 from Section IV.2 of Hairer and Wanner's book https://purdue. primo.exlibrisgroup.com/permalink/01PURDUE_PUWL/uc5e95/alma99169167132701081.
 - (a) Check that if $N \colon \mathbb{R}^n \to \mathbb{R}$ obeys (N1), (N2), and (N3), then d(x, y) = N(x y) defines a metric on \mathbb{R}^n
 - (b) Check that if $N: M_{n \times m} \to \mathbb{R}$ is the operator norm $N(A) = \max\{|Av|: v \in \mathbb{R}^m, |v| = 1\}$, then (N1), (N2), and (N3) hold.
- 2. Taylor 2.1.2, 2.1.3, 2.1.3A, 2.1.4 (https://mtaylor.web.unc.edu/wp-content/uploads/ sites/16915/2018/04/analmv.pdf). For 2.1.2 and 2.1.3, omit proving that H and Φ are C^1 , and for 2.1.4, omit the generalization to \mathbb{C}^n and \mathbb{C}^m .
- 3. Shifrin 3.3.11 (assume f is differentiable), 3.3.12 (omit the last question).

Hints:

1. This isn't a hint, but there are some other nice examples of norms in equations (1.7), (1.8), (1.9) of Section IV.1, with illustrations in Figures 1.4 to 1.7.

2.1.2. Since F is differentiable, we have F(X + Y) = F(X) + DF(X)Y + R(X,Y), where $||R(X,Y)||/||Y|| \to 0$ as $||Y|| \to 0$, (here $|| \cdot ||$ is as in equation (2.1.17)) and a similar expansion for G. Use these and the inequalities (2.1.18) to deduce the corresponding expansion for H.

3.3.12. Let h(t) = f(ta + (1 - t)b), where a and b are in U.