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MA 442
Spring 2024

Homework 3

Due February 1st on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. There are some hints on the second page.

1. Shifrin 3.2.15, 5.1.7, 9.4.11, 9.4.12ab.
2. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = \cos(x)\cos(2y)$. Find the Hessian of f as a function of (x,y). What is the Hessian at a point where f attains its global minimum?
3. Find the Hessian at $(0,0)$ of $f(x,y) = \sin\log(1+x-y+xy)$
4. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and define $f, g\colon \mathbb{R}^2 \to \mathbb{R}$ by $f(x) = (Ax)\cdot x$ and $g(x) = |x|^2$. Find all points where $f(x)$ is maximized subject to the constraint $g(x) = 1$, and compute $\nabla f(x)$ and $\nabla g(x)$ at those points. Do we have $\nabla f(x) = \lambda \nabla g(x)$ and $Ax = \lambda x$?

Hints:

3.2.15. Prove that if $v \in \mathbb{R}^n$ and $g(t) = f(a + tv)$, then $g'(0) = 0$.

5.1.7. Proposition 4.5 of Chapter 1 says that $(Av) \cdot w = w^T Av$. It may be helpful to let v be a vector such that $\|A\| = |Av|$ and then choose w so that $|Av| = (Av) \cdot w$.

9.4.11 Take a unit vector $v \in \mathbb{R}^m$, write it in terms an orthonormal basis of eigenvectors, and plug into $|Av|^2 = (A^T Av) \cdot v$.

2. See Taylor equation (2.1.59) and the Example on the following page.

3. Use the series of sin and log at 0, discarding terms of order 3 and higher.