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MA 442
Spring 2024

Homework 6

Due March 7th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

1. and 2. Exercises 1 and 2 from `https://www.math.purdue.edu/~kdatchev/442/ode.pdf`.

3. Prove that geodesics have constant speed by showing that if

$$-g_{jk}x_j'' - \partial_\ell g_{jk} x_\ell' x_j' + \tfrac{1}{2}\partial_k g_{j\ell} x_j' x_\ell' = 0, \qquad \text{for } k = 1, \dots, n,$$

then $g_{jk}x_j'x_k'$ is independent of t.

Hint: Differentiate $g_{jk}x_j'x_k'$ with respect to t, use the product rule and chain rule, and then use the geodesic equation to prove that the two terms where the derivative lands on a x' factor are each equal to $(-1/2)$ times the term where the derivative lands on the g factor.