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MA 442
Spring 2021

Homework 7

Due March 17th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake.

1. Part (1) of Exercise 2.9 from `https://www.math.purdue.edu/~kdatchev/442/geodesics.pdf`
2. Let $R > 0$ and $h > 0$ be given. Use the parametrization $\varphi(\theta, z) = (R\cos\theta, R\sin\theta, z)$ of the cylinder of radius R to find the lengths of the two shortest geodesics from $(\theta, z) = (0, 0)$ to $(\theta, z) = (\pi/2, 0)$ and the lengths of the three shortest geodesics from $(\theta, z) = (0, 0)$ to $(\theta, z) = (\pi/2, h)$. Sketch the geodesics in the (θ, z) plane (and, if you want, also on the cylinder).
3. Prove that geodesics have constant speed, i.e. that if

$$-g_{jk}\ddot{x}^j - \partial_\ell g_{jk}\dot{x}^\ell\dot{x}^j + \tfrac{1}{2}\partial_k g_{j\ell}\dot{x}^j\dot{x}^\ell = 0, \qquad \text{for } k = 1, \ldots, n,$$

   then $g_{jk}\dot{x}^j\dot{x}^k$ is independent of t.
   Hint: Differentiate the expression with respect to t, and use the geodesic equation to prove that the two terms where the derivative lands on a $\dot{x}$ factor are each equal to $(-1/2)$ times the term where the derivative lands on the g factor.