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MA 442
Spring 2021

Homework 9

Due April 14th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake.

1. Exercise 1a of Section 3.2 from page 157 of `https://mtaylor.web.unc.edu/wp-content/uploads/sites/16915/2018/04/analmv.pdf`
2. Exercise 2 of Section 3.2 from page 157 of `https://mtaylor.web.unc.edu/wp-content/uploads/sites/16915/2018/04/analmv.pdf` . Here, $\chi_{[0,1]}$ is the function which is 1 on $[0,1]$ and 0 everywhere else. Use (3.2.38) to write your answer without the gamma function and check that it agrees with the familiar formulas when $n = 1, 2, 3$. (You don't have to hand this in but you may also like to check how the area of any sphere is the derivative with respect to the radius of the volume of the corresponding ball.)
3. Let $r_1, \cdots r_n$ be given positive numbers. Find the area of the surface in $\mathbb{R}^{2n}$ given by the system of n equations
$$x_1^2 + x_2^2 = r_1^2, \qquad \ldots \qquad x_{2n-1}^2 + x_{2n}^2 = r_n^2.$$
(This kind of surface is called an n-dimensional torus; 1-dimensional tori are very familiar, and another version of a 2-dimensional torus has appeared previously in this course.)
4. Let $a < b$ be real numbers and let $\gamma\colon (a,b) \to (0,\infty)$ be a C^∞ function and let $M = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 = \gamma(z)^2\}$. Find a formula for $\int_M f dS$, where f is a continuous function, in terms of f and γ.