

Homework 3

Due September 16th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

1. Exercise 15 from page 44.
2. Let X be a metric space and let $K \subset X$ be compact. Prove that K is bounded.
3. Let K be a compact metric space and $\varepsilon > 0$. Show that there exists $N \in \mathbb{N}$ such that every set of N distinct points in K includes at least two points with distance less than ε between them.
4. Let X be a metric space and $K_1 \supset K_2 \supset K_3 \supset \dots$ a sequence of compact subsets of X . Prove that if $U \subset X$ is an open set with $\bigcap_{n=1}^{\infty} K_n \subset U$, then there exists $N \in \mathbb{N}$ such that $K_N \subset U$.