

### Homework 9

Due December 2nd on paper at the beginning of class. Please let me know if you have a question or find a mistake.

There are some hints on the second page.

1. Let  $K: [0, 1] \times [0, 1] \rightarrow \mathbb{R}$  be continuous, let  $g_1, g_2, \dots$  be a sequence of continuous functions  $[0, 1] \rightarrow [-1, 1]$ , and for each  $n$  and  $x$  let

$$f_n(x) = \int_0^1 K(x, y)g_n(y)dy.$$

Prove that a subsequence of  $f_1, f_2, \dots$  converges uniformly on  $[0, 1]$ .

2. Exercise 16 from page 168.
3. Exercise 18 from page 168. You may assume  $f$  is real valued.
4. Exercise 20 from page 169.

Hints:

1. Use Theorem 7.25. To check its hypotheses, use Theorems 4.15 and 4.19.
2. Examine the proof of Theorem 7.25.