MA 504 second midterm review problems

Version as of October 28th.

The second midterm will be in class on Monday October 31st. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Please let me know if you have a question or find a mistake.

1. Show that if $\sum a_n$ converges absolutely, then $\sum a_n^4$ does too. Give an example showing this does not hold for conditional convergence.

2. Let $f\colon [0,\infty) \to \mathbb{R}$ be continuous and such that

$$\lim_{x\to\infty} f(x) = 37.$$

   Prove that f is bounded.

3. Let $f\colon (-10,1) \to \mathbb{R}$ be continuous, such that

$$\lim_{x\to 1} f(x) = +\infty,$$

   and such that f is decreasing on $(-10,-5)$. Show that f has a minimum on $(-10,1)$.

4. Exercise 14 from page 100.

5. For which values of k is $f(x) = x^k$ uniformly continuous on $[1,\infty)$? If it is uniformly continuous, given any $\varepsilon > 0$ find $\delta > 0$ such that $|f(x) - f(y)| < \varepsilon$ when $|x - y| < \delta$. If it is not, find $\varepsilon > 0$ and sequences x_n and y_n in $[1,\infty)$ such that $|x_n - y_n| < 1/n$ and $|f(x_n) - f(y_n)| \geq \varepsilon$.

6. Let $a \neq 0$ be given, and let $n \geq 2$ be an even integer. Prove that $x = 0$ is the only solution to $x^n + a^n = (x+a)^n$.

   Hint: Argue by contradiction and use the mean value theorem.