MA 510 final review problems

Hopefully final version as of April 28th.

The final will be a take-home exam assigned Monday, May 4th, by 10:30am and due Wednesday, May 6th, by 12:30pm. It will be about twice as long as one of the midterms. It will cover all the material from the whole semester. While taking the exam, you may look up any information you need, but you may not use any calculating devices or programs or discuss the problems with anyone but me. Most of the problems on the exam will be closely based on ones from the first two exams, from the review problems of the first two exams, and from the list below. For each problem, you must justify your answers. Please let me know if you have a question or find a mistake.

1. Let S be the surface given by $x^2 + y^2 + 2z^2 = 1$, $x \geq 0$, $y \geq 0$, $z \geq 0$.
   (a) Find the area of S.

   (b) Find the flux of $(z, 0, 0)$ through S, oriented away from the origin.

2. Find a constant a such that for any region D in the xy plane, the surface area of the graph above D of $f(x, y) = x^2 + y^2$ matches the surface area of the graph above D of $g(x, y) = axy$.

3. Sketch the region given by the inequalities $x^2 + y^2 + z^2 \leq 4$, $x^2 + y^2 \leq 1$, and find its volume and surface area.

4. Evaluate $\int_C (z^2 + yz\sin(xyz))dx + (y^2 + xz\sin(xyz))dy + (x + xy\sin(xyz))dz$ where C is the curve following the outline for the triangle from $(1, 0, 0)$ to $(0, 1, 0)$ to $(0, 0, 1)$ and back to $(1, 0, 0)$.

5. Find the flux of $(y^2 z, (x + 1)^z, 0)$ through the surface given by $x = y^2$, $0 \leq z \leq 3$, $x \leq 8$, oriented towards the x axis.

6. Let
$$F = \frac{(x, y, z)}{(x^2 + y^2 + z^2)^{3/2}}.$$
   Use the fact that $\nabla \cdot F = 0$ when $(x, y, z) \neq (0, 0, 0)$ to find the flux of F through the following surfaces.
   (a) The cylindrical shell given by $x^2 + y^2 = 25$, $|z| \leq 5$, oriented outward.

   (b) The conical shell given by $x^2 + y^2 + (z - 2)^2 = 1$, $0 \leq z \leq 1$, oriented downward.

   (c) The square with corners $(1, 1, 1)$, $(1, -1, 1)$, $(-1, 1, 1)$, and $(-1, -1, 1)$, oriented

upward.
Hint: Use the fact that this is a face of a cube centered at the origin.