

### Homework 4

Due February 12th at the beginning of class, or by 12:30 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. Let  $f(x, y) = x^2 + y^2$ . Find  $\nabla f$  at the point  $(1, 2)$ , and the tangent line to the level set at that point. Then sketch the level set, gradient, and tangent line on a single graph.
2. Let  $f(x, y) = x^2 + y^2 + z^2$ . Find  $\nabla f$  at the point  $(1, 2, 3)$ , and the tangent plane to the level set at that point. Then sketch the level set, gradient, and tangent plane on a single graph.
3. Consider the surface parametrized by  $(x, y, z) = (uv + 1, \sin u + \cos(\pi v), u^2 - v)$ .
  - (a) For which  $u$  and  $v$  does the surface pass through  $(1, -1, -1)$ ?
  - (b) Find an equation for the tangent plane to the surface at  $(1, -1, -1)$ .
4. Let  $x(y, z)$  be given implicitly by the equation

$$\sin(3x - 2y + z) + x + y^2 + z^3 = 0,$$

near  $(0, 0, 0)$ . Find  $\partial_y x(0, 0)$  and  $\partial_z x(0, 0)$ .

5. For each of the functions below, find the matrix of second partial derivatives at the origin, and decide if the critical point there is a local maximum, a local minimum, or neither.
  - (a)  $f(x, y) = \cos(x^2 + 2y^2)$
  - (b)  $f(x, y) = xy$
  - (c)  $f(x, y, z) = xy + z^2$
  - (d)  $f(x, y, z) = x^2 + 2y^2 + yz + 2z^2$