Kiril Datchev
MA 510
Spring 2020

Homework 7

Due March 11th at the beginning of class, or by 12:30 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. Use Leibniz' rule for differentiating under the integral (see `https://www.math.purdue.edu/~kdatchev/510/leibniz.pdf`) to simplify the following:

   (a)
   $$\frac{d}{dt}\int_0^1 \frac{e^{ty^2}}{y}dy.$$

   (b)
   $$\frac{d}{dt}\int_{t^2}^{t^3} \frac{e^{ty^2}}{y}dy.$$

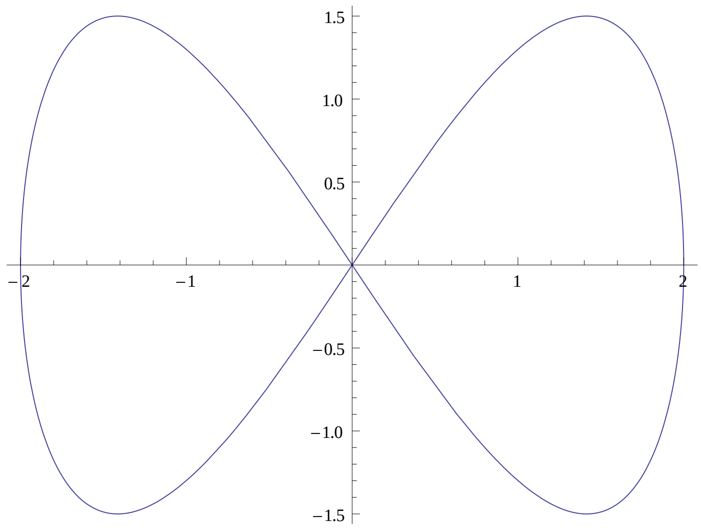
2. Evaluate
   $$\int_C 2(y + x\sin y)dx + x^2\cos y dy,$$
   where C is the oriented outline of the parallelogram from $(1,1)$ to $(1,4)$ to $(2,5)$ to $(2,2)$ and back to $(1,1)$.

3. Evaluate
   $$\int_C 2y\sin^2 x dx - (x + \sin x\cos x)dy,$$
   where C is the oriented parabolic curve from $(2,2)$ to $(2,0)$ given by $x = 2y^2 - 4y + 2$.

   Hint: It may be helpful to use Green's Theorem for the region given by $0 \le y \le 2$ and $2y^2 - 4y + 2 \le x \le 2$.

4. The parametric curve
   $$x(t) = 2\sin t, \quad y(t) = 3\sin t\cos t, \qquad 0 \le t \le 2\pi,$$
   is pictured on the next page. Find the area inside the curve.

   Hint: It may be helpful to use Green's Theorem in the form
   $$\iint_D dxdy = -\int_C ydx,$$
   with a carefully chosen region D. It may be easier to take D which is only part of the total region!



1.5
1.0
0.5
−0.5
−1.0
−1.5
−2
−1
1
2