

Homework 9

Due April 8th by 12:30pm by email to kdatchev@purdue.edu. Justify your answers. Please let me know if you have a question or find a mistake.

1. The equations

$$\begin{aligned}x(t) &= \cos t, \\y(t) &= \sin t,\end{aligned}$$

for $0 \leq t \leq 2\pi$ parametrize the circle $x^2 + y^2 = 1$.

(a) Modify the equations so that they instead parametrize the ellipse

$$x^2 + 4y^2 = 9.$$

(b) Find constants a and b such that the arc length of the ellipse is given by

$$\int_0^{2\pi} \sqrt{a \cos^2 t + b \sin^2 t} dt$$

2. The equations

$$\begin{aligned}x(u, v) &= \cos u \sin v, \\y(u, v) &= \sin u \sin v, \\z(u, v) &= \cos v,\end{aligned}$$

for $0 \leq u \leq 2\pi$ and $0 \leq v \leq \pi$ parametrize the sphere $x^2 + y^2 + z^2 = 1$.

(a) Modify the equations so that they instead parametrize the ellipsoid

$$x^2 + 4y^2 + 9z^2 = 16.$$

(b) Find constants a , b , and c such that the surface area of the ellipsoid is given by

$$\int_0^{2\pi} \int_0^\pi \sin v \sqrt{a \cos^2 u \sin^2 v + b \sin^2 u \sin^2 v + c \cos^2 v} dv du.$$