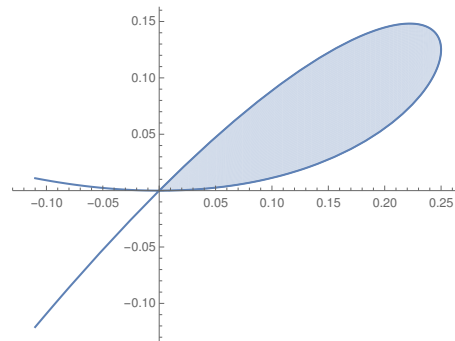
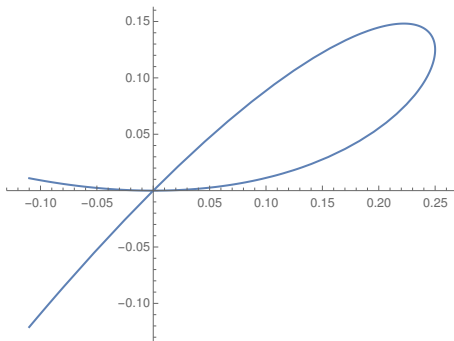


Midterm 2

Due April 2nd by 12:30 pm by email. You may look up any information you need, but you may not use any calculating devices or programs or discuss the problems with anyone but me. Justify your answers. Please let me know if you have a question or find a mistake.

1. (a) Which of the following vector fields can be written as ∇f for some function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$? If it can be done, find such a function f .
 - i. $(0, x)$,
 - ii. $(x^2 + y, x)$,
 - iii. $(x^2 + y, -x)$,
- (b) Choose one of the vector fields from part (a) and evaluate its line integral along the circular arc starting at $(1, 0)$, passing through $(2, 1)$, and ending at $(1, 1)$.
2. Below on the left is a graph of the parametric curve $x(t) = t(1-t)$ and $y(t) = t^2(1-t)$. On the right is the same curve with a region shaded. Find the area of the shaded region.



3. Simplify the following:

(a)

$$\frac{d}{dt} \int_{-t}^t \frac{\sin(ty)}{y} dy.$$

(b)

$$\frac{d}{dt} \int_{-t}^t \left[\frac{\sin(ty)}{y} + y \sin \cos \ln \left(1 + t^2 y^2 + (1 + t^2)^{1+t^2} \right) \right] dy.$$

4. Let D be the region given by $1 \leq \frac{1}{4}x^2 + 9y^2 \leq 2$ and $0 \leq -\frac{1}{2}x \leq 3y$. Find

$$\iint_D y(x^2 + 36y^2)^{-1/2} dx dy.$$

5. Let W be the region given by $\sqrt{x^2 + y^2} \leq z$, $x^2 + y^2 + z^2 \leq 4$, and $x^2 + y^2 + (z - \frac{1}{2})^2 \geq \frac{1}{4}$. Find

$$\iiint_W \frac{z}{x^2 + y^2 + z^2} dx dy dz.$$