MA 530 final review problems
Version as of April 25th.

The final exam will be in HAAS G066 from 3:30 to 5:30 on Thursday, May 4th. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. Prove that there exists $R > 0$ such that the function $z \mapsto \dfrac{z^2}{1 - e^{z^2}}$ is holomorphic and bounded on the set $\{z \in \mathbb{C} \colon 0 < |z| < R\}$.

2. Let $f \colon \mathbb{C} \to \mathbb{C}$ be holomorphic and suppose that $|f(z)| \le 2 + 3|z|$ for all $z \in \mathbb{C}$. Prove that f is a polynomial, and find all possible values of the degree of f and of $f(1)$.

3. Let $f(z) = z^5 + 20z^2 + 22$. For each positive integer n, determine how many zeroes of f lie in the set $A_n = \{z \in \mathbb{C} \colon n - 1 < |z| < n\}$.

4. Evaluate and simplify
$$\int_0^\infty \frac{x^{2/3}}{(x+2)^2 + 4} dx.$$

5. Let $\alpha \in (0, \pi)$, let $\Omega = \{z \in \mathbb{C} \colon |\arg z| < \alpha\}$, and let $p \in \Omega$. Find all biholomorphisms $f \colon \Omega \to D = \{z \colon |z| < 1\}$ such that $f(p) = 0$.

6. Let $f \colon \{z \colon |z| < 1\} \to \{z \colon \operatorname{Re} z > 1\}$ be a holomorphic function such that $f(0) = 2$. Find a number A such that $\operatorname{Re} f(1/2) \le A$.

7. Let $f \colon \{z \colon \operatorname{Im} z > 0\} \to \mathbb{C}$ be holomorphic. Suppose $f(1+i) = 3 - i$ and $\operatorname{Im} f(z) = 0$ when $|z| = 1$. For which p is $f(p)$ determined by the above information, and what is $f(p)$?

8. # 1 from `https://www.math.purdue.edu/files/academic/grad/qualexams/MA53000/MA53000_2022_JAN.pdf`

9. # 6 from `https://www.math.purdue.edu/files/academic/grad/qualexams/MA53000/MA53000_2022_Aug.pdf`